Info

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# Left-invariant vector fields

$$g(-) : G \to G$$

is a diffeomorphism of $G$.

**Definition.** Left-invariant vector fields

Let $G$ be a Lie group. A vector field $X \in \mathrm{Vec}(G)$ is called **left-invariant** if

$$g, h \in G \implies \mathrm{d}(g(-))(X_h) = X_{gh}$$

or, equivalently

$$g \in G \implies g(-)_* X = X$$

and the set of all such vector fields are denoted by $\mathrm{leftVec}(M)$.

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and if

$$X, Y \in \mathrm{leftVec}(M) \implies [X, Y] \in \mathrm{leftVec}(M)$$

$$g \in G \implies g(-)_*[X, Y] = [g(-)_* X, g(-)_* Y] = [X, Y]$$

Let $G$ be a Lie group. The evaluation map

$$\begin{aligned} \mathrm{eval}_i : \mathrm{LVec}(M) &\to T_i G \\ X &\mapsto X_i \end{aligned}$$

is a $\mathbb{R}$-linear isomorphism.

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Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold group (Lie groups)
      - Lie algebra of a lie group
        - Left-invariant vector fields

And it has 2 siblings.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold group (Lie groups)
      - Lie algebra of a lie group
        - Exponential map from lie algebra of a lie group
        - Left-invariant vector fields