

This note, that was found here, as a part of a collection is written (completely with human hands) by Rupadarshi Ray, was created on March 4, 2026 9:46:01 PM, was last modified on September 7, 2026 1:21:16 PM, and was exported on September 7, 2026 3:09:55 PM.

 $\longleftrightarrow$

flat tori

$$n = 2$$

quotients of $L := GL(2)(\mathbb{R}) / GL(2)$	homeo	moduli of lattices/flat tori	$\mathbb{Z}$-basis
L	4-manifold?	all lattices/tori	
$\frac{L}{O_+(2, \mathbb{R})}$	$\mathbb{R}^3$	upto oriented isometries	
$\frac{L}{CO_+(2, \mathbb{R})} \cong? \frac{H}{SL(2)(\mathbb{R})}$	$\mathbb{C} \cong \mathbb{R}^2$	upto oriented conformal maps	$v_1 = (1, 0)$ and $v_2 \in F$
$\frac{L}{O(2, \mathbb{R})}$	$[0, \infty) \times \mathbb{R}^2?$	upto isometries	12.5. CLASSIFICATION OF FLAT TORI: Let $T^2 = S^1 \times S^1$ denote the 2-torus. Show that if g is a flat Riemannian metric on T^2, then (T^2, g) is isometric to one and only one Riemannian quotient S^2/Λ, where Λ is a lattice generated by a basis of the form $\{(a, 0), (b, c)\}$, with $a > 0, 0 \leq b \leq a/2, c > 0$, and $b^2 + c^2 \geq a^2$. [Hint: Given a lattice $\Lambda \subseteq \mathbb{R}^2$, let v_1 be an element of $\Lambda \setminus \{0, 0\}$ of minimal norm; let v_2 be an element of $\Lambda \setminus \{v_1\}$ of minimal norm (where $\{v_1\}$ is the cyclic subgroup generated by v_1), chosen so that the angle between v_1 and v_2 is less than or equal to $\pi/2$, and then apply a suitable orthogonal transformation.] [1]
$\frac{L}{CO(2, \mathbb{R})} \cong? \frac{H/SL(2, \mathbb{R})}{\mathbb{Z}_2}$	$[0, \infty) \times \mathbb{R}$	upto (unoriented) conformal maps	1/2 (if not $a_1 = a_2$ would be shorter than a_2). The class of the lattice Γ is hence determined by the position of a_2 in the hatched domain $At = \{(x, y) x^2 + y^2 \geq 1, 0 \leq x \leq 1/2, y > 0\}$, and two lattices corresponding to two different points of At belong to two different classes.

1. <https://www.desmos.com/calculator/l6fgnoug6> ↩

Current note has 0 direct children and 0 total descendants.

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And it has 39 siblings.

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