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Suspension of diffeomorphisms of smooth manifolds

[1]

Definition. Suspension flow of a diffeomorphism on the mapping torus

Let

$$f : M \rightarrow M$$

be a diffeomorphism of a smooth manifold M to itself.

On the *mapping torus of the diffeomorphism*

$$M_f := \frac{M \times [0, 1]}{(x, 0) \sim (f(x), 1)}$$

along with the projection

$$\pi : M_f \rightarrow M$$

we consider the flow

$$\begin{aligned} \phi^\theta : M_f &\rightarrow M_f \\ (x, t) &\mapsto (x, t + \theta \bmod 1) \end{aligned}$$

called the **suspension flow** of the diffeomorphism f .

- The vector field corresponding to the suspension flow is

$$\frac{d}{d\theta} \Big|_{\theta=0} \phi^\theta = \frac{\partial}{\partial t} \text{ on } TM_f$$

which is along $T\mathbb{R}$ in the split $TM_f = TM \oplus T\mathbb{R}$.

So we observe the *information* about the flow or the vector field is in the manifold M_f itself.

- The unit time map

$$\begin{aligned} \phi^1 : M_f &\rightarrow M_f \\ (x, t) &\mapsto (x, t + 1 \bmod 1) \end{aligned}$$

- Now

$$\begin{array}{ccc} M_f & \xrightarrow{\phi^1} & M_f \\ (x, t) & & (x, t + 1 \bmod 1) \\ \downarrow \pi & & \downarrow \pi \\ M & \xrightarrow{f} & M \\ x & & f(x) \end{array}$$

- Functor?

$$\begin{array}{ccc} \text{End}(\text{Man}) & \rightarrow & \text{FlowMan} \\ M & & \\ \downarrow f & \mapsto & (M_f, \frac{\partial}{\partial t}) \\ M & & \end{array}$$

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