Info

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Measurable functions on $\mathbb{R}^n$

Definition. Measurable functions on $\mathbb{R}^n$

Let $E \subseteq \mathbb{R}^n$ be a measurable subset. A function $f : E \to \mathbb{R}$ is **measurable** if

$$f^{-1}(-\infty, a) = \{f < a\}$$

is measurable for all $a \in \mathbb{R}$.

Let

$$f : \mathbb{R}^d \to [0, \infty)$$

be a measurable function. Then there exists a sequence of simple functions $\{\varphi_n : \mathbb{R}^d \to [0, \infty)\}$ such that

$$\phi_n \underset{n\to\infty}{\nearrow} f$$

Consider the N-th truncation of f

$$F_N := f 1_{[0,N]^d \cap \{f \leq N\}} + N 1_{[0,N]^d \cap \{f > N\}}$$

which is bounded

$$F_N : \mathbb{R}^d \to [0, N]$$

has support $\subseteq [0, N]^d$ and converges to f pointwise

$$F_N \xrightarrow{N\to\infty} f$$

- Now we parition the codomain $[0, N]$ of F_N into MN parts and approximate F_N by the simple function

$$F_{N,M} := \sum_{0 \leq l \leq NM} \frac{l}{M} 1_{\{\frac{l}{M} < F_N \leq \frac{l+1}{M}\}}$$

- This implies

$$0 \leq F_N - F_{N,M} \leq \frac{1}{M}$$

- Then

$$\phi_k := F_{2^k, 2^k} \underset{n\to\infty}{\nearrow} f$$

(Ergorov) Let (f_n) be a sequence of measurable function on $E \subseteq \mathbb{R}^d$ with $m(E) < \infty$ and

$$f_n \to f \text{ aeon } E$$

Then for any given $\epsilon > 0$ we can find a closed set $A_\epsilon \subset E$ such that $m(E - A_\epsilon) \leq \epsilon$ and

$$f_n \xrightarrow{\text{uniformly}} f \text{ on } E$$

(Lusin) Let f be measurable function on $E \subseteq \mathbb{R}^d$ with $m(E) < \infty$. Then for every $\epsilon > 0$ there exists a closed set $F_\epsilon \subset E$ such that $m(E - F_\epsilon) \leq \epsilon$ so that

$$f\Big|_{F_\epsilon}$$

is continuous.

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - Measurable functions on $\mathbb{R}^n$

And it has 15 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - Functions with uniformly bounded mean oscillations on cubes
      - Decomposition of $f \in L^p$ into bounded and unbounded parts
      - Calderon-Zygmund decomposition
      - Lebesgue density of measurable sets
      - Functions with uniformly bounded mean oscillations on dyadic cubes
      - Volterra operator $\int_{[0,-]} : L^1_{\text{loc}}[0,1) \to \mathcal{C}[0,1)$
      - Measurable functions on $\mathbb{R}^n$
      - (ϵ, n)-measurable function

- Integrability and integral of measurable functions on $\mathbb{R}^n$
- Hardy-Littlewood maximal functions of $L^1_{\text{loc}}(\mathbb{R}^n)$
- Lebesgue averaging and differentiation
- Integrals of a monotonically converging sequence of functions
- Lebesgue integral from measure of undergraph
- $L^{p,q}$
- $\mathrm{E}([0,1]) = \mathrm{E}(0,1), \mathrm{E}([-\pi,\pi]) = \mathrm{E}(S^1)$