Info

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Arc length

Definition. Arc length

Given a parameterized curve γ the **arc length** between the points $\gamma(t_1)$ and $\gamma(t_2)$ is

$$\mathfrak{L}[\gamma](t_1, t_2) := \int_{[t_1,t_2]} \|\gamma'\|$$

Arc length remains invariant under space isometries.

The arc length transforms like

$$\mathfrak{L}[\gamma](t_1, t_2) = \mathfrak{L}[\tilde{\gamma}](\theta(t_1), \theta(t_2))$$

where

$$\tilde{\gamma} = \gamma \circ \theta.$$

Now we take the arc length function as

$$\mathfrak{L}[\gamma](t, t_0) := \int_{[t_0,t]} \|\gamma'\|$$

for some arbitrary t_0. Hence, from fundamental theorem of analysis on $\mathbb{R}$ we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathfrak{L}[\gamma](t, t_0) = \|\gamma'\|$$

If γ is regular, arc length function $\mathfrak{L}[\gamma](t)$ is smooth.

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And it has 6 siblings.

- stem
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