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Conic neighborhood

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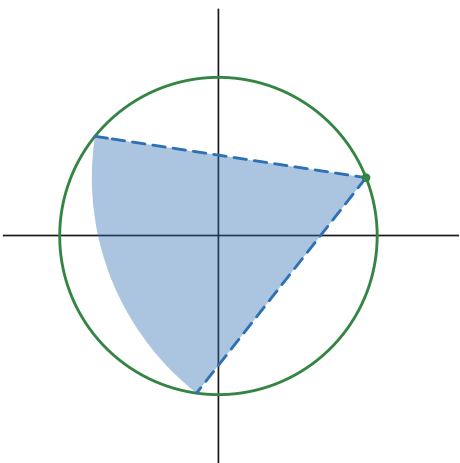
Definition.

$$I(a, k, \alpha) := \left\{ x \in S^{n-1} \left| \left| x - \frac{a}{|a|} \right| < k(1 - |a|)^\alpha \right. \right\} \\ = B_{k(1-|a|)^\alpha} \left(\frac{a}{|a|} \right) \cap S^{n-1}$$

✓ <https://www.desmos.com/3d/url9lns0su>

Definition. Cone in a ball

$$\text{cone}(\xi, \lambda) := \left\{ x \in B^n \left| \angle_0(\xi, \xi - x) < \lambda, \cos \lambda > \frac{|x - \xi|}{2} \right. \right\} \\ = \left\{ x \in B^n \left| \frac{|\xi - x|^2 + 1 - |x|^2}{2|\xi - x|} > \cos \lambda > \frac{|x - \xi|}{2} \right. \right\}$$



✓ <https://www.desmos.com/calculator/ten8dlusbu>

✓ <https://www.desmos.com/3d/wjg534hj3a>

convergence in conic neighbourhoods

Proposition:

Let $\xi \in S^{n-1}$ and $\{x_n\} \subseteq B^n$ be a sequence such that $|x_n| \xrightarrow{n \rightarrow \infty} 1$. Then the following are equivalent

1. $\exists a > 0$ such that

$$\forall n \gg 1, x_n \in \text{cone}(\xi, a)$$

2. $\exists b > 0$ such that for

$$\forall n \gg 1, |x_n - \xi| \leq b(1 - |x_n|)$$

3. $\exists c > 0$ such that

$$\forall n \gg 1, \xi \in I(x_n, c, 1)$$

4. $\exists d > 0$ such that for any hyperbolic geodesic γ ending at ξ we have

$$\forall n \gg 1, d_{\mathbb{R}H^n}(x_n, \gamma) < d$$

We have

$$b(o, B_r(x)) := \\ = \{ \xi \in \partial \mathbb{R}H^n | d(x, \xi) < r \} \\ = \left\{ \xi \in S^{n-1} \left| \begin{array}{l} |x - \xi| |x + \xi| < (1 - |x|^2) \cosh r \\ |x + \xi| > |a - \xi| \end{array} \right. \right\}$$

5. Thus $\exists r$ such that

$$\forall n \gg 1, \xi \in b(o, B_r(x_n))$$

Proposition:

$$\cosh d(x, \overline{\xi\eta}) = \frac{2}{|\xi - \eta|} \frac{|a - \xi| |a - \eta|}{1 - |a|^2}$$

Definition.

$$L(a, k, \alpha) := \bigcap_{N \geq 1} \bigcup_{n > N} I(\gamma_n(a), k, \alpha)$$

• $\alpha > 0 \implies L(a, k, \alpha) \subseteq \Lambda(\Gamma)$

Proposition: Let Γ converges at $(n - 1)\alpha$. Then

$$m_{S^{n-1}} \left(\bigcup_{k > 0} L(a, k, \alpha) \right) = 0$$

Current note has 0 direct children and 0 total descendants.

- [squishy](#)
 - $\mathbb{R}\boldsymbol{H}^n$
 - [Conic neighborhood](#)

And it has 5 siblings.

- [squishy](#)
 - $\mathbb{R}\boldsymbol{H}^n$
 - [Hyperbolic ball model](#)
 - [Conic neighborhood](#)
 - [Hyperboloid model of \$\mathbb{R}\boldsymbol{H}^n\$](#)
 - [Hyperboloid model to ball model of \$\mathbb{R}\boldsymbol{H}^n\$](#)
 - [Types of isometries of \$\mathbb{R}\$ -hyperbolic space](#)