ⓘ Info

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e^{-x^2}

Fourier transform on $\mathbb{R}$

Given

$$\varphi : \mathbb{R} \to \mathbb{R}$$
$$\varphi(x) := e^{-\pi x^2}$$

consider the function

$$F : \mathbb{R} \to \mathbb{C}$$
$$F(s) := \int_{\mathbb{R}} e^{-\pi(t+is)^2}$$

For $h \in [-1,1]$ by fundamental theorem of calculus we write

$$\frac{F(s+h)-F(s)}{h} = -2\pi i \int_{t\in\mathbb{R}} \int_{\theta\in[0,1]} e^{-\pi(t+is+ih\theta)}(t+is+ih\theta)$$

and passing the $h \to 0$ limit inside the integrals by LDCT? we have F is differentiable and

$$F'(s) = -2\pi i \int_{t\in\mathbb{R}} (t+is)e^{-\pi(t+is)^2 \mathrm{d}t} = 0$$

So F is constant and is equal to $F(0) = 1$.

Now the Fourier transform of φ on $\mathbb{R}$ is obtained by

$$1 = \int_{t\in\mathbb{R}} e^{-\pi(t+is)^2} = e^{\pi s^2}\hat{\varphi}(s)$$

Thus

$$\hat{\varphi}(s) = e^{-\pi s^2} = \varphi(s)$$

[1]

on $\mathbb{C}$

$$\exp(-z^2)$$

Current note has 0 direct children and 0 total descendants.

- squishy
  - pow
    - e^{-x^2}

And it has 28 siblings.

- squishy
  - pow
    - Angle function
    - Bessel functions
    - $$\cos\sqrt{z}$$
    - The complex logarithm
    - $\sum_{n\geq 0} z^{n!}$
    - $z^3 - 3z$
    - $$\frac{r_0}{1+\epsilon\cos b\theta}$$
    - $\cos\left(\pi\frac{p}{q}\right)$
    - $\frac{\cos x^k}{x}$
    - Elliptic integrals and Jacobi elliptic functions
    - The complex exponential $\exp(z)$
    - $\exp\left(\frac{z+1}{z-1}\right)$
    - e^{-x^2}
    - [Only sum terms that are multiples of N in $\mathbb{C}[[z](https://rupadarshiray.github.io/notes/YchnxQ9SF1RsuKjBntMv8a.$]]
    - $\int \frac{1}{x^2+bx+c}$
    - $\frac{\exp(z)}{1-z}$
    - $\sin x$
    - $\sin(x\sin(x))$
    - $\frac{\sin x}{x}$
    - stock.sin x by xk
    - $\sin x^2$
    - $\sqrt{z}$
    - $\sqrt{z^2-1}$
    - Theta functions
    - Weierstrass elliptic function $\wp$
    - Weierstrass' primary factors
    - $x^2\sin\frac{1}{x}$
    - $x + x^2\sin\frac{1}{x}$

1. Loukas Grafakos - Fundamentals of Fourier Analysis - 2024.pdf > page=66 ↩