Info

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Generating symplectomorphisms

Definition. Generating functions of symplectomorphisms

For any $f \in \mathcal{C}^\infty(M_1 \times M_2)$, $\mathrm{d}f$ is a closed 1-form on $M_1 \times M_2$. Then the Lagrangian submanifold generated by f in $T^*(M_1 \times M_2)$ is the graph Y_f of $\mathrm{d}f$. Let

$$\mathrm{d}_{(x,y)} f = \mathrm{d}_x f + \mathrm{d}_y f$$

using the decomposition

$$T^*_{(x,y)}(M_1 \times M_2) = T^*_x X \oplus T^*_y X_2$$

then

$$Y_f = \{(x, y, \mathrm{d}_x f, \mathrm{d}_y f) \mid (x, y) \in M_1 \times M_2\}$$

and also define

$$Y_f^\sigma := \{(x, y, \mathrm{d}_x f, -\mathrm{d}_y f) \mid (x, y) \in M_1 \times M_2\}$$

When Y_f^σ is graph of a diffeomorphism $\phi : T^*M_1 \to T^*M_2$, we call it the **symplectomorphism generated by** f and call f the **generating function** of ϕ.

Now Y_f^σ is graph of $\phi : T^*M_1 \to T^*M_2$ if and only if

$$(x, y, \phi(x, y)) = (x, y, \mathrm{d}_x f, -\mathrm{d}_y f)$$

Locally, in coordinates (x_i, α_j) on T^*M_1 and (y_i, β_j) on T^*M_2 we have

$$\phi(x_i, \alpha_j) = (y_i, \beta_j) \iff \alpha_j = \mathrm{d}_x f, \beta_j = -\mathrm{d}_y f$$

[1]

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - ℝ-manifold symplectic
      - Generating symplectomorphisms

And it has 8 siblings.

- structures based on sets
  - manifolds
    - ℝ-manifold symplectic
      - Symplectic capacities
      - Compatible almost complex structure on a symplectic manifold
      - Generating symplectomorphisms
      - Symplectic homogeneous manifolds
      - Quantization of symplectic manifolds
      - Lagrangian submanifolds of a symplectic manifold
      - Symplectic Lie group action on a symplectic manifold
      - Hamiltonian vector fields on a symplectic manifold

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1. Ana Cannas da Silva - Lectures on Symplectic Geometry -Springer (2009).pdf > page=35 ↩