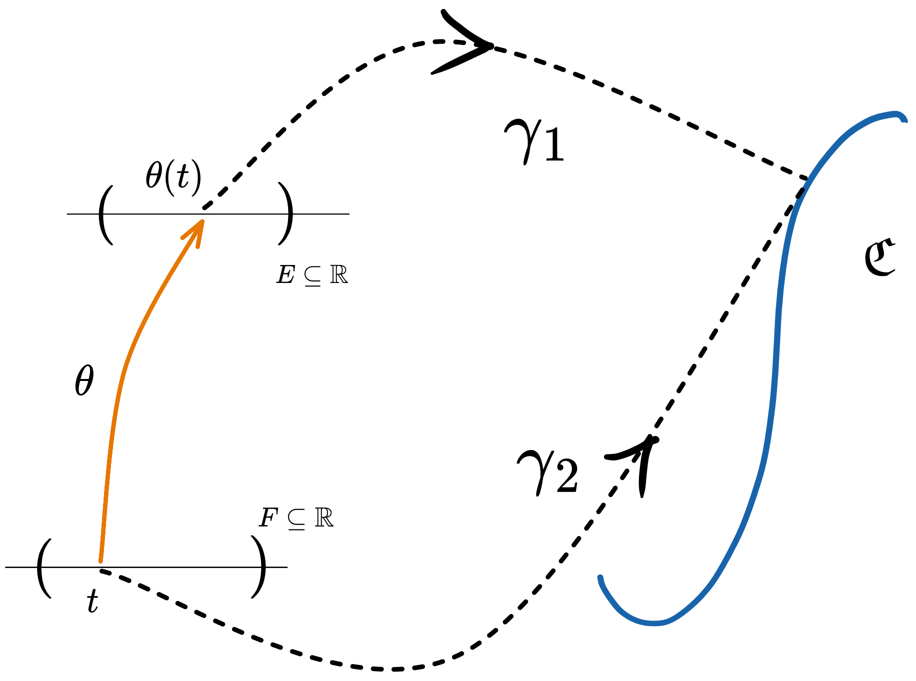
Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on September 3, 2022 4:37:05 PM,
was last modified on September 7, 2026 1:10:55 PM, and
was exported on September 7, 2026 3:09:52 PM.

Reparameterization of a *parameterized* curve

Definition. Reparameterizations of *parameterized* curves



Given two parameterized curves $\gamma_1 : E \to \mathbb{R}^n, \gamma_2 : F \to \mathbb{R}^n$, then γ_2 is called a **reparameterization** of γ_1 if there exists a *diffeomorphism* $\theta : F \to E$ such that

$$\gamma_2(t) = (\gamma_1 \circ \theta)(t),\ \forall t \in E$$

This θ is a *"coordinate transformation"* where the previous coordinate was t and the final coordinate is $\theta(t)$.

orientation

Any diffeomorphism θ is strictly monotonic,

$$\theta' > 0 \text{ or } < 0$$

Thus we can define a measurement of the sign of this change.

Definition. Orientation preservation

$$\alpha_O[\theta] := \frac{\theta'}{|\theta'|} = \begin{cases} 1 \text{ if } \theta' > 1 \\ -1 \text{ if } \theta' < 1 \end{cases}$$

If $\alpha_O[\theta] = 1$, then θ is called a **orientation preserving transformation**, and when it is -1, it is **orientation reversing**.

Obviously, $\alpha_O^2 = 1$ for any θ. This is an important property to check because

$$(\gamma \circ \theta)'(t) = (\gamma' \circ \theta)(t)\ \theta'(t)$$

so, orientation preserving transformations keep the direction of velocity the same, and orientation reversing, reserves the direction of velocity.

Set of all diffeomorphisms of a curve

$$CT[\text{curve } \mathfrak{C}] := \{\theta \mid \theta \text{ is a diffeomorphism of } \mathfrak{C}\}$$

is a group under composition of function

$$(CT[\text{curve } \mathfrak{C}], \circ)$$

and *reparameterization* is a group action on set of all parameterizations of the curve.

The transformations include:

- Ones that preserve speed at every point

$$\theta(t) = \pm t + t_0$$

- Ones that scale velocity by λ

$$\theta(t) = \lambda t$$

parameterization properties

The quantities that change with parameterization in general

- **"velocity"**
  - $\gamma' \to (\gamma' \circ \theta)(\theta')$
- **"acceleration"**
  - $\gamma'' \to (\gamma'' \circ \theta)(\theta')^2 + (\gamma' \circ \theta)(\theta'')$

geometric properties

The quantities that change with parameterization in a particular way

- Arc length
- Tangent unit of a curve

- $$\hat{\mathbf{T}}[\gamma \circ \theta] = \alpha_O[\theta]\,(\hat{\mathbf{T}}[\gamma] \circ \theta)$$

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma\backslash\mathbb{R}^n, \Gamma\backslash\mathbb{C}^n$
    - Curves
      - Reparameterization of a parameterized curve

And it has 6 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma\backslash\mathbb{R}^n, \Gamma\backslash\mathbb{C}^n$
    - Curves
      - Arc length
      - Congruent curves
      - Curvature of a curve
      - Reparameterization of a parameterized curve
      - Tangent of a curve
      - Unit speed parameterization of a curve