Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on April 28, 2024 12:06:29 PM,
was last modified on September 7, 2026 1:11:00 PM, and
was exported on September 7, 2026 3:09:54 PM.

Ramified germs of smooth and holomorphic functions

Definition. Ramified germs of holomorphic functions

With the stalk of holomorphic functions at $0 \in \mathbb{C}^n$, $\mathscr{O}_0(\mathbb{C}^n) \equiv \mathscr{O}_n$ and the ideal $\mathfrak{m}_n \subseteq \mathscr{O}_n$ of such functions vanishing at 0, let the group of germs(?) of biholomorphisms $\mathscr{D}_n$ act on $\mathscr{O}_n$ by $f \overset{g}{\mapsto} f \circ g^{-1}$. Then

- a **critical point** at 0 is a germ $f \in \mathscr{O}_n$ such that $f'(0) = 0$
- a **singularity** is a equivalence class $[f] \in \mathscr{O}_n/\mathscr{D}_n$

Definition. Stable equivalence of germs

The germs $f \in \mathfrak{m}_{n_1}, g \in \mathfrak{m}_{n_2}$ are **stably equivalent** if

$$f + x_{n_1+1}^2 + \cdots + x_k^2$$

is equivalent to g

$$\frac{\coprod_{n \geq 1} \mathfrak{m}_n}{\text{stable equivalence}}$$

Definition. Versal deformations of a critical point

A **deformation with base $\mathbb{C}^l$ of the germ** $f \in \mathscr{O}_n$ is the germ at $0 \in \mathbb{C}^n \times \mathbb{C}^l$ of a smooth map

$$F : (\mathbb{C}^n \times \mathbb{C}^l, 0) \to \mathbb{C},\ F(-, 0) = f$$

- two deformations F_1, F_2 are **equivalent** if

  $$F_1(x, \lambda) = F_2(g_\lambda(x), \lambda)$$

  where $g_\lambda : (\mathbb{C}^n \times \mathbb{C}^l, 0) \to (\mathbb{C}^n, 0)$ is a smooth germ of $\lambda \in \mathbb{C}^n$ parameter family of diffeomorphisms
- the deformation F_2 is **induced** from F_1 is

  $$F_2(x, \lambda) = F_1(x, \theta(\lambda))$$

  where $\theta : (\mathbb{C}^{l_1}, 0) \to (\mathbb{C}^{l_2}, 0)$ is a smooth germ of a mapping of the bases
- deformation F of a germ f is said to be **versal** if every deformation F' of f can be represented of the form

  $$F'(x, \lambda) = F(g(x, \lambda), \theta(\lambda)),\ g(x, 0) = x,\ \theta(0) = 0$$

  that is, every deformation of f is *equivalent* to a deformation *induced* from f

Deformations are "perturbations".

Definition. Modality of a critical point

The group of germs of diffeomorphisms act on the function space $\mathfrak{m}_n \subset \mathscr{O}_n$ of functions $f : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$. The modality $\mathrm{md}(f)$ of such a function germ is the modality of any of its jets $j^k f, k \geq \mu(f) + 1$.
Modality is the minimum k such that a sufficiently small nbd of f is covered by a finite number of k-parameter families of orbits.

Does this number answer how many *inequivalent "perturbations"* of f are there?

Definition. Level bifurcation set of a singularity

classification of singularities of holomorphic functions upto stable equivalence

- Simple singularities of holomorphic functions

modality $= 1$			
modality $= 2$			

Current note has 1 direct children and 1 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Ramified germs of smooth and holomorphic functions
      - Simple singularities of holomorphic functions

And it has 39 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - Open subsets of $\mathbb{R}^n$ with $\mathcal{C}^2$ boundary

- Complex ODEs in $\mathbb{C}$
- Circle packing on $\mathbb{R}^2$
- Circle packing converges to the Riemann biholomorphism
- Bounded connected open subsets of $\mathbb{C}^n$
- Connected circular open subsets of $\mathbb{C}^n$
- Continuous functions on $\mathbb{R}^d$
- Dyadic cubes
- Curves
- Differentiable functions
- Differential forms on $\mathbb{R}^n$
- Fourier-Wigner transform
- Harmonic functions composed with conformal maps
- Hilbert transform
- Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
- Holomorphic subsets of $\mathbb{C}^n$
- Regular hypersurfaces in $\mathbb{R}^{2n}$
- Orientable hypersurfaces in $\mathbb{R}^n$
- Laplace operator on $\mathbb{R}^n$
- $l^\infty(\mathbb{R})$
- Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
- Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
- Local $\mathbb{R}$-algebras
- $l^p(\mathbb{R})$
- Metric density of subsets of $\mathbb{R}^n$
- Monotone functions on $\mathbb{R}$
- Cauchy integral of periodic functions
- Polygons with integer vertices
- Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
- Discrete subgroups of $\mathbb{R}^n$
- Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- Ramified germs of smooth and holomorphic functions
- Open subsets of $\mathbb{R}^n$
- Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- Closed subsets of $\mathbb{R}^n$ equipped with smooth functions
- Quasi-analytic smooth functions on $\mathbb{R}$
- Star-shaped subsets of $\mathbb{R}^n$
- ODEs in $\mathbb{R}^n$ $\leftrightarrow$ Vector fields in $\mathbb{R}^n$