Info

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S_3

It is smallest non-Abelian group.

$$S_3 \cong \langle \tau, s | \tau^3 = i = s^2, s\tau s = \underbrace{\tau^{-1}}_{\tau^2} \rangle$$

where τ will map to a 3-cycle and s, a 2 cycle.

$\text{FinVec}_{\mathbb{C}}$ representations

- Permutation representation of S_3
- Standard representation of S_3 (irreducible, dim $= 2$)

But to describe any arbitrary irreducible representation we look at

$$\mathbb{Z}_3 \cong_{\text{Grp}} \underbrace{\{i, \tau, \tau^2\}}_{\text{3-cycles}} < S_3$$

Any arbitrary finite dim representation $S_3 \to GL_{\mathbb{C}}(V)$ must have an action of the Abelian subgroup

$$\{i, \tau, \tau^2\} \cong \mathbb{Z}_3 \to GL_{\mathbb{C}}(V)$$

is a representation, so it must break into one dimensional subspaces, implying eigenvectors $(v_i)_{i=1}^n$ of τ span V whose eigenvalues must be

$$\tau v_i = \lambda_i v \implies \tau^3 v_i = v = \lambda_i^3 v \implies \lambda_i = \omega^{\alpha_i}$$

for some $\alpha_i \in \{1, 2, 3\}$ for each i where

$$V = \bigoplus_{i=1}^{n} \mathbb{C} v_i$$

Now

$$s\tau s = \tau^2 \implies \tau s = s\tau^2$$

then

$$\tau s(v_i) = s\tau^2(v_i) = \omega^{2\alpha_i} s(v_i)$$

So $s(v_i)$ is an eigenvector of τ with eigenvalue $\omega^{2\alpha_i}$

- if $\omega^{\alpha_i} \neq 1$
    - the eigenvalues

      $$\omega^{\alpha_i} \neq \omega^{2\alpha_i}$$

      so $v_i, s(v_i)$ are linearly independent

      $$\mathbb{C} v_i + \mathbb{C} s(v_i)$$

      is thus 2 dimensional S_3-invariant subspace of V, implying it is all of V. This is isomorphic to the standard representation of S_3
- if $\omega^{\alpha_i} = 1$
    - if $s(v_i) \in \mathbb{C} v_i$
        - $\mathbb{C} v_i$ is isomorphic to the *trivial* representation if $s(v_i) = v$
        - $\mathbb{C} v_i$ is isomorphic to the *alternating* representation if $s(v_i) = -v$

character table of representations

parition of 3	conjugacy classes of S_3	trivial	alternating	standard
$1+1+1$	1-cycle i	1	2	2
$2+1$	2-cycles	1	-1	2
3	3-cycles (1 2 3)	1	2	-1

Current note has 2 direct children and 2 total descendants.

- stretchy
    - Symmetric group
        - S_3
            - Permutation representation of S_3
            - Standard representation of S_3

And it has 5 siblings.

- stretchy
    - Symmetric group
        - S_3
        - S_4
        - S_5
        - S_n
        - $\lim S_n$