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Vector bundles over topological spaces

Definition. topological k -vector bundle E over topological space X

$$E \rightarrow X$$

A **topological k -vector bundle** $E \rightarrow X$ over a topological space X is a tuple

- a *topological space* E
- a *surjective continuous map*

$$\pi_E : E \rightarrow X$$

such that

- for each $x \in E$, there are maps $+_x, \cdot_x$ such that

$$(E_x := \pi^{-1}(x), +_x, \cdot_x)$$

is a k -vector space

- and it is *locally trivial*
 - for each $x \in X$, $\exists(U, n, \phi_U)$ a neighborhood U , $n \in \mathbb{N}_{\geq 0}$ and a homeomorphism

$$\phi_U : U \times k^n \rightarrow \pi^{-1}(U)$$

- such that for each $b \in U$, the map $k^n \rightarrow E_b$

$$z \mapsto (b, z)$$

is an $\mathbf{Vec}_k$ -isomorphism

Given object $E \in \mathbf{Top}(X)$ which is the [category over some object](#) $X \in \mathbf{Top}$ such that ^[1]

- $E \rightarrow X$ is a (*finite dim*) *vector space object inside the category* $\mathbf{Top}(X)$ (with the fiber product over X) (denote the category of all such objects as $\mathbf{Top}(X)\mathbf{Vec}_k?$)
- is locally isomorphic (in $\mathbf{Top}(X)$) to the trivial bundle

Then, E is a vector bundle over X .

Rank of a vector bundle

Definition. Morphism of vector bundles over topological spaces

Definition. Category of vector bundles over topological spaces

$$\mathbf{Vec}_k\mathbf{Bun}_k\mathbf{Top}$$

- **objects:** vector bundles $E \rightarrow X$ of rank k
- **morphisms:** vector bundle morphisms

Definition. Pullback of vector bundles



$$\mathbf{Vec}_k\mathbf{Bun}_k\mathbf{Top}_{\cong}(X) \in \mathbf{Set}$$

Definition.

$$\mathbf{Vec}_k\mathbf{Bun}_k\mathbf{Top}_{\cong} : \mathbf{Top}^{op} \rightarrow \mathbf{Set}_*$$

- **objects:**

$$X \mapsto (\mathbf{Vec}_k\mathbf{Bun}_k\mathbf{Top}_{\cong}(X), X \times k^k)$$

- **morphisms:**

First projection $X \times [0, 1] \rightarrow X$ through the functor $\mathbf{Vec}_k\mathbf{Bun}_k\mathbf{Top}_{\cong}$ induces an isomorphism

$$\mathbf{Vec}_k\mathbf{Bun}_k\mathbf{Top}_{\cong}(X) \rightarrow \mathbf{Vec}_k\mathbf{Bun}_k\mathbf{Top}_{\cong}(X \times [0, 1])$$



$$\mathbf{Vec}_k\mathbf{Bun}_k\mathbf{Top}_{\cong}(X) : \mathbf{hTop}^{op} \rightarrow \mathbf{Set}_*$$

- cat
- t - types/extension
 - rank
 - ...
- p - constructing more/attachments/constructed with
 - vector space operations
- m - structures/more structure
 - reducing the structure group

- v - points/insides/properties

- s - subsets
- f - functions/equations(polynomials, equations, differential equations)/dynamics
 - sections
- b - function spaces, attachments, bundles and duals
 - section spaces
 - $\mathbb{R}$ -vector space
 - $\mathcal{C}^0(X, \mathbb{R})$ -module
 - sheaf

- o - external effects
- e - equivalences
 - equivalences and automorphisms
 - isom classes
 - identifications
- c - list/examples/classification upto isom

Current note has 0 direct children and 0 total descendants.

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And it has 29 siblings.

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$$G \curvearrowright P \rightarrow X$$
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 - [Locally compact and 2-countable Hausdorff topological spaces are paracompact with countable refinement](#)
 - [Covering dimension of topological spaces](#)
 - $\text{EndTop}(X, Y)$
 - [Eilenberg–Steenrod functors](#)

$$(H_n : \text{Top} \times \text{Top} \rightarrow \text{Ab}, \partial_n)$$
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