ⓘ Info

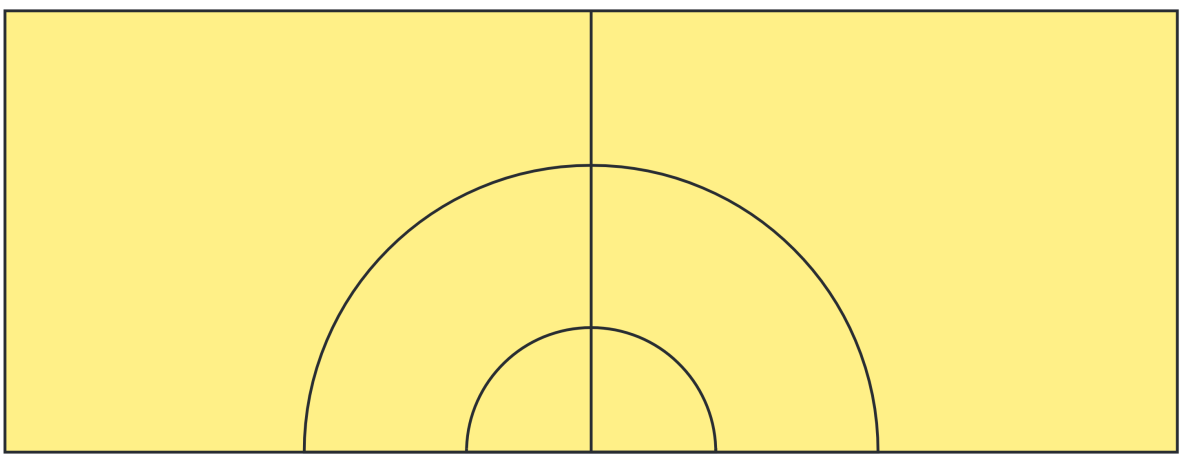
This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on June 29, 2026 2:23:15 PM,
was last modified on June 29, 2026 4:02:22 PM, and
was exported on September 7, 2026 2:50:36 PM.

Side pairings in $\mathbb{R}\boldsymbol{H}^2$

of two lines with no common endpoints

Say we have two geodesic lines

$$\overline{AB}, \overline{CD}$$



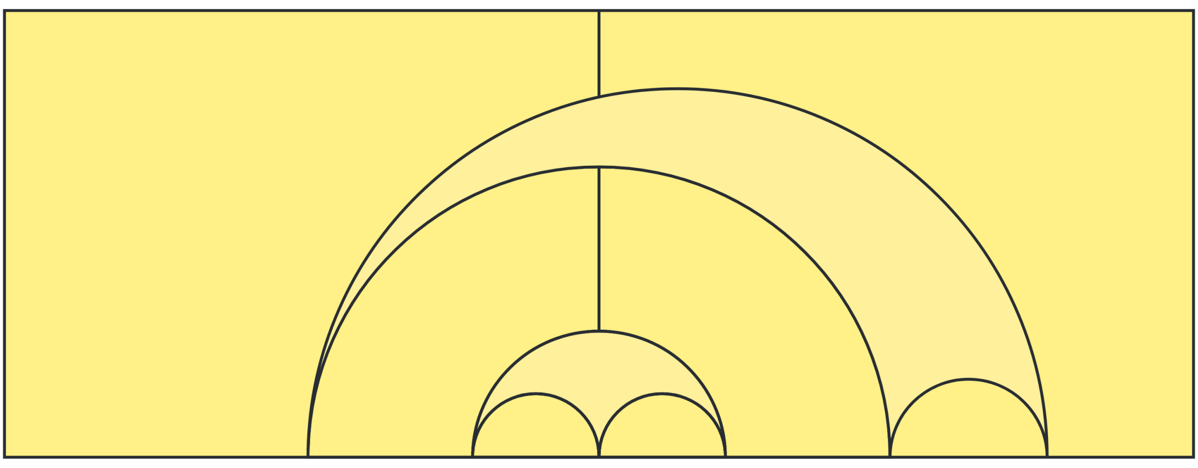
To list all elements in

$$\left\{g \in PSL(2)(\mathbb{R}) \middle| \overline{AB} \overset{g}{\mapsto} \overline{CD}\right\}$$

we can use the exact 3-transitivity of $PSL(2)(\mathbb{R}) \curvearrowright \partial\mathbb{R}\boldsymbol{H}^2$. Consider the triangles

$$\triangle(A, B, o), \triangle(C, D, \alpha)$$

where $\alpha \in \partial\mathbb{R}\boldsymbol{H}^2 \setminus \{C, D\}$ is arbitrary.

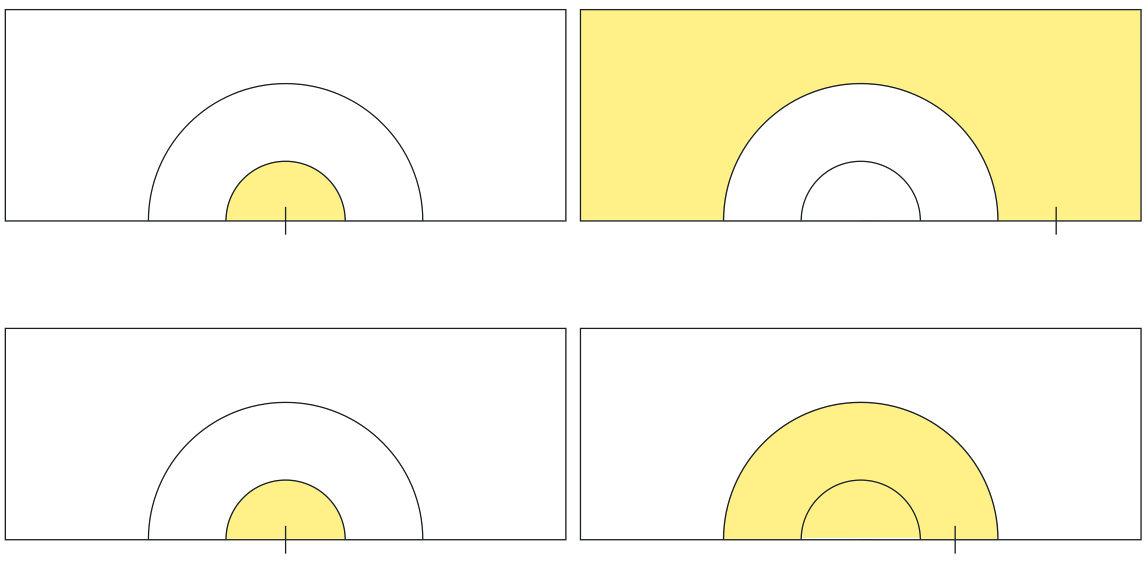


Then there is an unqiue $g_E \in PSL(2)(\mathbb{R})$ such that

$$A, B, o \overset{g_E}{\longmapsto} C, D, \alpha$$

For different values of $\alpha \in \partial\mathbb{R}\boldsymbol{H}^2 \setminus \{C, D\}$ we have different images of o under g_α.

The two connected components of $\partial\mathbb{R}\boldsymbol{H}^2 \setminus \{C, D\}$ represent the two possibilities where the two components of $\mathbb{R}\boldsymbol{H}^2 \setminus \overline{AB}$ map onto.



Now similarly consider $g'_E \in PSL(2)(\mathbb{R})$ such that

$$A, B, o \overset{g'_E}{\longmapsto} D, C, \alpha$$

Therefore, the subset $\left\{g \in PSL(2)(\mathbb{R}) \middle| \overline{AB} \overset{g}{\mapsto} \overline{CD}\right\}$ is just

$$\{g_\alpha, g'_\alpha | \alpha \in \partial\mathbb{R}\boldsymbol{H}^2 \setminus \{C, D\}\}$$

an one parameter four connected compotent collection of maps.

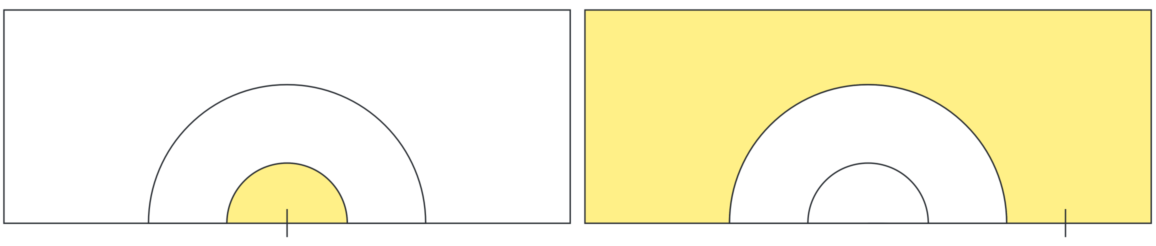
? Question

Classify $\langle g_\alpha \rangle, \langle g'_\alpha \rangle$ upto $PSL(2)(\mathbb{R})$-conjugation.

However, not all of these maps are suitable to define side pairings, that should satisfy the *tesselation* property

$$g_\alpha(P) \cap P = \overline{CD}$$

Because



leaves us with

$$\{g_\alpha, g'_\alpha | a \in (C, D)\}$$

However, g'_α also does not satisfy the *tesselation* property.

Current note has 0 direct children and 0 total descendants.

- squishy
    - $\mathbb{R}\boldsymbol{H}^2$, $\mathbb{R}H^2_{\mathbb{U}}$, D^2
        - Side pairings

And it has 2 siblings.

- squishy
    - $\mathbb{R}\boldsymbol{H}^2$, $\mathbb{R}H^2_{\mathbb{U}}$, D^2
        - Polygons in hyperbolic plane
        - Side pairings