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Topological spaces

Motivation

- [Metric spaces](#)
 - Continuous functions and neighborhoods*
- Graphs

Definition. Topology on a set X

Let X be a set. A **topology $\mathcal{T}$ on X** is a subset

$$\mathcal{T} \subseteq 2^M$$

such that

- $\emptyset \in \mathcal{T}, M \in \mathcal{T}$
- $U_\alpha \in \mathcal{T} \implies \bigcup_{\alpha \in A} U_\alpha \in \mathcal{T}$
- $V, U \in \mathcal{T} \implies U \cap V \in \mathcal{T}$

Definition. Topological spaces $(X, \mathcal{T})$

A **topological space $(X, \mathcal{T})$** is a set X and a *topology $\mathcal{T}$ on X* .

Definition. Topology on a set X

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