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Riemannian manifolds with non-positive sectional curvature *AKA* Cartan-Hadamard manifolds

(Cartan-Hadamard) Let (M, g) be a complete Riemannian manifold with non-positive sectional curvature $\kappa_g \leq 0$ then for any $p \in M$ the exponential map

$$\exp_p : T_p M \rightarrow M$$

is a **covering map**. In particular if M is simply connected then $\exp_p$ is a diffeomorphism.

universal cover

Proposition: Let (M, g) be a complete Riemannian manifold. Then the following are equivalent

- 1. every geodesic in (M, g) is (globally) **length-minimizing**
- 2. every unit speed geodesic in (M, g) is a minimal geodesic
- 3. every pair of points in (M, g) is joined by a **unique** minimal geodesic segment
- 4. the exponential

$$\exp_p : T_p M \rightarrow M$$

is a diffeomorphism for all $p \in M$

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