Info

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Unit tangent bundle of a Riemannian manifold

Definition. Unit tangent bundle of a Riemannian manifold

Let (M, g) be a Riemannian manifold. Then

$$T^{\mathrm{u}}M := \left\{ X_p \in TM \middle| \|X_p\|_{\mathrm{g}} = 1 \right\}$$

is the **unit tangent bundle** of (M, g).

contact form

Proposition: The 1-form

By the diffeomorphism

$$\begin{aligned} \mathrm{g} : TM &\to T^*M \\ X_p &\mapsto \mathrm{g}_p(X_p, -) \end{aligned}$$

we have the 1-form

$$\begin{aligned} T_u TM &\to \mathbb{R} \\ (V + H) &\to \mathbb{R} \\ \alpha_u(v + w) &:= \mathrm{g}_{\pi(u)}(u, v) \end{aligned}$$

on $T^{\mathrm{u}}M$ satisfies

$$\alpha \wedge (\mathrm{d}\alpha)^{\dim M - 1} \neq 0 \qquad \text{on } M$$

^DGRBookCurrentv2510, p.47

Current note has 0 direct children and 0 total descendants.

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    - $\mathbb{R}$-manifold semi-Riemannian
      - Unit tangent bundle of a Riemannian manifold

And it has 18 siblings.

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