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Compatible almost complex structure on a symplectic manifold

Definition. An almost complex structure J is **compatible** with a symplectic structure ω if

$$\omega(-, J-)$$

is a Riemannian metric.

Corollary 12.7. Any symplectic manifold has compatible almost complex structures.

– How different can compatible almost complex structures be?

Proposition 12.8. Let (M, ω) be a symplectic manifold, and J_0, J_1 two almost complex structures compatible with ω . Then there is a smooth family $J_t, 0 \leq t \leq 1$, of compatible almost complex structures joining J_0 to J_1 .

Proof. By compatibility, we get

$$\left. \begin{aligned} \omega, J_0 &\rightsquigarrow g_0(\cdot, \cdot) = \omega(\cdot, J_0 \cdot) \\ \omega, J_1 &\rightsquigarrow g_1(\cdot, \cdot) = \omega(\cdot, J_1 \cdot) \end{aligned} \right\} \quad \text{two riemannian metrics on } M .$$

Their convex combinations

$$g_t(\cdot, \cdot) = (1 - t)g_0(\cdot, \cdot) + tg_1(\cdot, \cdot) , \qquad 0 \leq t \leq 1 ,$$

form a smooth family of riemannian metrics. Apply the polar decomposition to (ω, g_t) to obtain a smooth family of J_t ’s joining J_0 to J_1 . □

Corollary 12.9. The set of all compatible almost complex structures on a symplectic manifold is path-connected.

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