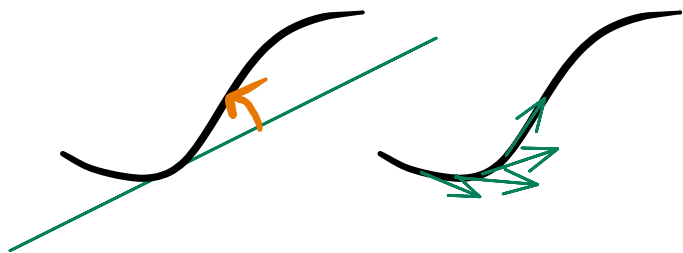


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Curvature of a curve

Motivation: “How much does a curve *curve*?”



be “measured” by how much the unit tangent vector changes with t .

Definition. Curvature $\kappa(t)$ of a curve

Given a *unit speed parameterization* of a curve $\tilde{\gamma}$, the

$$\kappa[\tilde{\gamma}](t) := \|\tilde{\gamma}''(t)\|$$

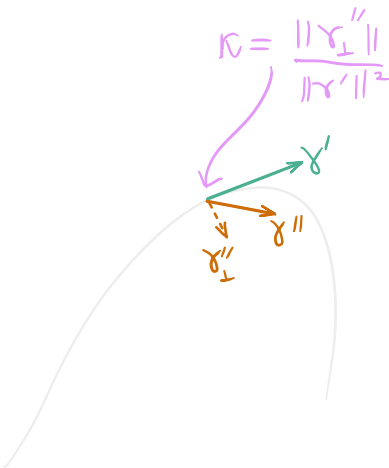
is the **curvature** of the curve at the point $\gamma(t)$.

Between any two unit-speed parameterizations, as acceleration stays invariant, curvature also remains invariant.

For any parameterization γ the curvature is

$$\kappa[\gamma](t) = \frac{\|\gamma'' - (\gamma'' \cdot \hat{\mathbf{T}})\hat{\mathbf{T}}\|}{\|\gamma'\|^2}(t)$$

The $\gamma'' - (\gamma'' \cdot \hat{\mathbf{T}})\hat{\mathbf{T}}$ term just takes the acceleration and removes the component of it parallel to velocity. Thus the curvature is just the norm of the component of acceleration perpendicular to velocity, divided by norm of velocity squared.



$$\gamma'' = \underbrace{\|\gamma'\|}_{\gamma''_{\parallel}} \hat{\mathbf{T}} + \underbrace{\|\gamma'\|}_{\gamma''_{\perp}} \hat{\mathbf{T}}'$$

$\gamma''_{\perp} \rightarrow \text{change of dir}$

Under any reparameterization, the curvature should remain invariant, because it is defined from the unit-speed parameterization, and it actually is:

≡

$$\kappa[\gamma \circ \theta] = \kappa[\gamma] \circ \theta$$

>

Current note has 0 direct children and 0 total descendants.

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 - [Curves](#)
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And it has 6 siblings.

- [stem](#)
 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n\$](#)
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