Info

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is written (completely with human hands) by Rupadarshi Ray,
was created on October 14, 2023 6:26:30 PM,
was last modified on April 3, 2025 7:06:25 PM, and
was exported on September 7, 2026 3:31:18 PM.

Poisson manifold

Definition. Poisson manifold

A **Poisson manifold** (M, B) is a smooth manifold M with a lie bracket

$$B : \mathcal{C}^\infty(M) \times \mathcal{C}^\infty(M) \to \mathcal{C}^\infty(M)$$

on the $\mathbb{R}$-vector space $\mathcal{C}^\infty(M)$ which satisfies the Leibniz identity

$$f, g, h \in \mathcal{C}^\infty(M) \implies B\{f, gh\} = B\{f, g\}h + gB\{f, h\}$$

This basically means the bracket is a *second exterior vector field*

$$\mathrm{B} \in \mathrm{Vec}^2(M)$$

Definition. Hamiltonian vector fields on a Poisson manifold

For any $H \in \mathcal{C}^\infty(M)$ on a Poisson manifold M, the **Hamiltonian vector field of** H is

$$B\{H, -\} : \mathcal{C}^\infty(M) \to \mathcal{C}^\infty(M)$$

$$X_H(H) = B\{H, H\} = 0$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold Poisson

And it has 20 siblings.

- structures based on sets
  - manifolds
    - Closed aspherical manifolds
    - $\mathbb{C}$-manifold
    - Hermitian manifolds
    - Kahler manifolds
    - Kahler manifolds of complex dimension 2, AKA Kahler surfaces
    - smooth $\mathbb{R}$-manifold
    - Principal bundles over smooth manifolds
    - Manifold with a flow
    - $\mathbb{R}$-manifold group (Lie groups)
    - Homogeneous manifold
    - Finsler manifold
    - $\mathbb{R}$-manifold semi-Riemannian
    - Sub-Riemannian manifolds
    - $\mathbb{R}$-manifold Poisson
    - $\mathbb{R}$-manifold symplectic
    - Multisymplectic manifolds
    - Spin $\mathbb{R}$-manifold
    - $\mathbb{R}$-manifold topological
    - Toric manifolds
    - Manifolds with compatible triples