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## When are closed differential 1-forms exact on a smooth manifold?

Because a conservative 1-form is closed, it is a necessary condition. But what are the conditions for a closed form to be conservative/exact? Or,

when is  $H^1_{dR}(M) = 0$  ?

### simply connected manifold implies all closed 1-forms are conservative

- In a simply connected manifold any closed curve *segment* is path-homotopic to a constant curve.

- Thus because

☰ In a smooth manifold  $M$ , let  $\gamma_1, \gamma_2 : [a, b] \rightarrow M$  be two path-homotopic piecewise smooth *curve segments* for every closed 1-form on  $M$

$$\int_{\gamma_1} \omega = \int_{\gamma_2} \omega$$

- taking any closed curve *segment*

$$\int_{\gamma} \omega = \int_{p \in M} \omega = 0$$

- Hence, every closed form is conservative.

☰ In a simply connected smooth manifold,



$$H^1_{dR}(M) = 0$$

This is a sufficient but not necessary condition.

## When are closed differential 1-forms *not* exact on a smooth manifold?

when  $\oint \omega \neq 0$

☰ Given a closed 1-form  $\omega$ , it is not exact/non-conservative  $\iff \exists$  a piecewise smooth closed curve  $\gamma$  such that

$$\oint_{\gamma} \omega \neq 0$$

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- [structures based on sets](#)
  - [manifolds](#)
    - [smooth  \$\mathbb{R}\$ -manifold](#)
      - [Tangents on a smooth manifold](#)
        - [Differential forms on smooth manifolds](#)
          - [Differential 1-forms on smooth manifolds](#)
            - [When are closed differential 1-forms exact on a smooth manifold?](#)

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        - [Differential forms on smooth manifolds](#)
          - [Differential 1-forms on smooth manifolds](#)
            - [Closed differential 1-forms on smooth manifolds](#)
            - [When are closed differential 1-forms exact on a smooth manifold?](#)
            - [Conservative 1-forms](#)