Info

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$\mathscr{O} \cap L^2(D)$

Let $f \in \mathscr{O} \cap L^2(D)$ with

$$f(z) = \sum_{n\in\mathbb{N}} a_n z^n \qquad \text{on } D$$

then

$$\begin{aligned}\|f\|_2^2 &= \int_D |f^2| \\ &= \int_D f\overline{f} \\ &= \int_D \sum_{n,m\in\mathbb{N}} a_n\overline{a_m} z^n \overline{z^m} \\ &= \sum_{n\in\mathbb{N}} |a_n|^2 \int_D |z|^{2n} \\ &= \pi \sum_{n\in\mathbb{N}} \frac{|a_n|^2}{n+1}\end{aligned}$$

$$\hat{} : \mathscr{O} \cap L^2(D) \to l^2\left(\mathbb{N}, \frac{1}{\mathrm{Id}+1}\right)$$

is an isometric isomorphism.

[1]

Proposition:

$$\mathscr{O} \cap L^2(D) = \mathscr{O} \cap L^2(D^\times)$$

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - $\mathscr{O}(D)$
        - $\mathscr{O} \cap L^2(D)$

And it has 5 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - $\mathscr{O}(D)$
        - Boundary values of $\mathscr{O}(D)$
        - $\mathscr{O}(\overline{D})$
        - $\mathscr{O}(D) \cap \mathcal{C}(\overline{D})$
        - $\mathscr{O} \cap L^2(D)$
        - Reconstructing $\mathscr{O}(D)$ from its boundary values

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1. https://math.stackexchange.com/a/2286797 ↩