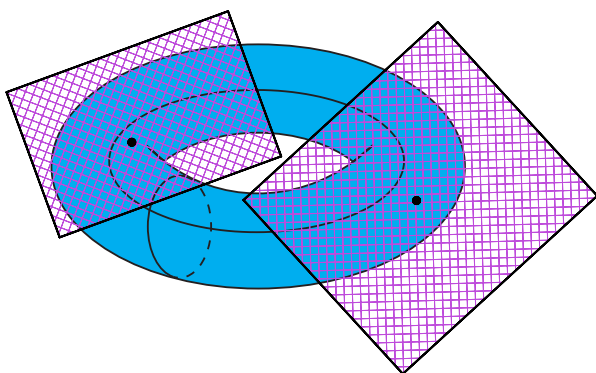


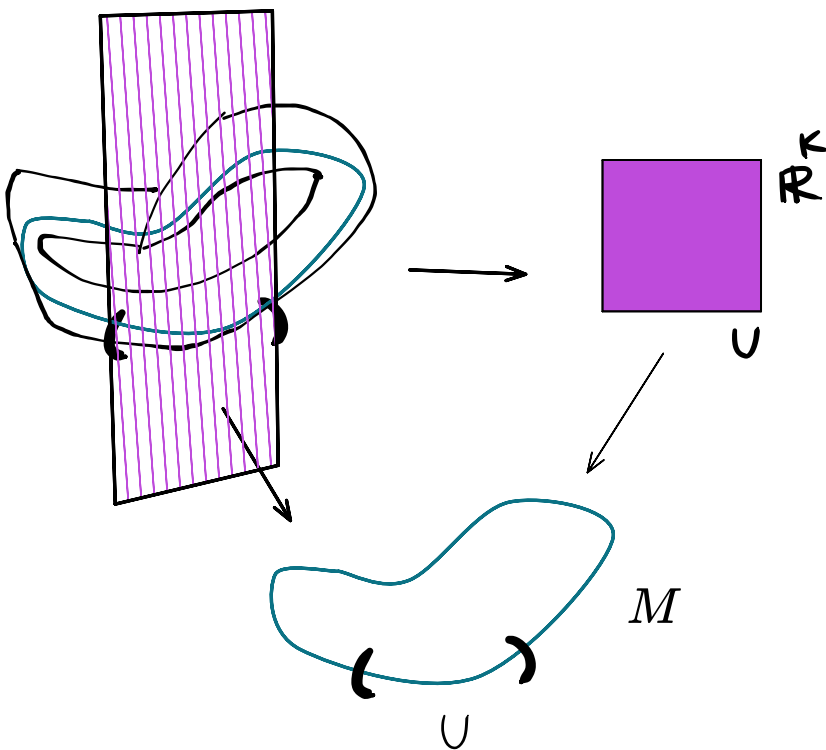
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Vector bundle on smooth manifolds



Definition. Vector bundle on smooth manifolds

A $\mathbb{R}$ -vector bundle of rank k over M



is a

$\pi_E : E \rightarrow M$

such that

- E_p is a $\mathbb{R}$ -vector space of dimension k
- for each $p \in M$,
 - there exists neighbourhood U of p
 - and a homeomorphism $\Phi : \text{preim}_{\pi_E}(U) \rightarrow U \times \mathbb{R}^k$
 - such that both projects on on U
 - and $\Phi|_{E_p}$ is a [vector space isomorphism](#)

$E_p \cong_{\text{Vec}_{\mathbb{R}}} \mathbb{R}^k$

- cat
 - $\text{ManVec}_{\mathbb{R}} \mathcal{B}$
- t - types/extension
- p - constructing more/attachments/constructed with
 - vector operations
 - $E_1 \otimes E_2$
 - $E_1 \oplus E_2$
 - E^* dual bundle
 - $\Gamma(E^*)$
 - $\text{HomVec}_{\mathbb{R}}(E_1, E_2) \cong E_1 \otimes E_2^*$
 - $T^{p,q} E$
 - $\Gamma(T^{p,q} E) =: \Gamma T^{p,q}(E)$
 - $\Lambda^k E$
 - $\Gamma(\Lambda^k E) =: \Gamma \Lambda^k(E)$
 - $\bigoplus_k \Gamma(\Lambda^k E) =: \Gamma \Lambda^\bullet(E)$
 - $(\Gamma \Lambda^\bullet(E), +, \cdot, \wedge)$
 - $\Lambda^\bullet E$?
 - frame bundle $\mathcal{F} E$
- m - structures/more structure
 - reducing the structure group
 - sections of some bundle
 - metrics
 - induction [Inducing vector bundles](#)
 - [Homotopic maps induce isomorphic vector bundles](#)

- v - points/insides/properties
- s - subsets

	• subbundles
• f - functions/equations(polynomials, equations, differential equations)/dynamics	• <i>sections</i>
• b - function spaces, attachments, bundles and duals	• <i>sections space</i> $\mathbf{\Gamma}(E)$ <i>E</i>-vector fields • $\mathbb{R}$-vector space $(\mathbf{\Gamma}(E), +, \cdot)$ • $\mathcal{C}^\infty$-module $(\mathbf{\Gamma}(E), +, \cdot)$ • sheaf $U \mapsto \mathbf{\Gamma}_U(E)$
	• cohomology of vector bundles

• o - external effects	
• e - equivalences	• equivalences and automorphisms • identifications
• c - list/examples/classification upto isom	
• isom classes	$\mathbf{Vec}_{\mathbb{F}}\mathbf{BunTop}(M)$ $\mathbf{Vec}_{\mathbb{F}}\mathbf{BunMan}(M)$ • $\mathbf{Vec}_{\mathbb{F}}\mathbf{Bun}_k\mathbf{Man} : \mathbf{Man}^{op} \rightarrow \mathbf{Set}_*$ • functor $\mathbf{VB}_k(-)$ • <u>Vector bundles over a contractible manifold</u>

• <u>Connection on a smooth vector bundle on a smooth manifold</u>
• <u>Pullback of a smooth vector bundle on a smooth manifold</u>

Current note has 5 direct children and 6 total descendants.

• <u>structures based on sets</u>	
• <u>manifolds</u>	• <u>smooth $\mathbb{R}$-manifold</u> • <u>Vector bundle on smooth manifolds</u> • <u>Connection on a smooth vector bundle on a smooth manifold</u> • <u>Inducing vector bundles</u> • <u>Homotopic maps induce isomorphic vector bundles</u> • <u>Vector bundles over a contractible manifold</u> • <u>Pullback of a smooth vector bundle on a smooth manifold</u> • <u>Tangent bundle of a smooth vector bundle</u>

And it has 19 siblings.

• <u>structures based on sets</u>	
• <u>manifolds</u>	• <u>smooth $\mathbb{R}$-manifold</u> • <u>Smooth Lie group action on a smooth manifold</u> • <u>Borel measures on a smooth manifold</u> • <u>G-bundles over smooth manifolds</u> $G \curvearrowright P \rightarrow X$ • <u>Vector bundle on smooth manifolds</u> • <u>Curves in smooth manifolds</u> • <u>Density on smooth manifolds</u> • <u>Functions between smooth manifolds</u> • <u>Functions from smooth manifolds to $\mathbb{R}$</u> • <u>Flows on a smooth manifold</u> • <u>Locally $G \curvearrowright X$-manifolds</u> • <u>Smooth singular homology of a smooth manifold</u> • <u>Isomorphisms and automorphisms of smooth manifolds</u> • <u>Lagrangian flows</u> • <u>Local transformation rules</u> • <u>Moduli space of Riemannian metrics on a smooth manifold</u> • <u>Moduli space of quotients</u> • <u>Quotient manifolds</u> • <u>Submanifolds of smooth manifolds</u> • <u>Tangents on a smooth manifold</u>