Info

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# $\mathbb{C}\setminus\{0,1\}$, $PSL(2)(\mathbb{Z})[2]\backslash\mathbb{R}\boldsymbol{H}^2_{\mathrm{U}}$

#wiki/Man/C/1

- diffeomorphic to Punctured plane $\mathbb{R}^2\setminus\{0\}$

> **Proposition:** For any $a\neq b\in\mathbb{C}$ we have
> $$\mathbb{C}\setminus\{a,b\}\cong_{\mathsf{Man}_{\mathbb{C}}}\mathbb{C}\setminus\{0,1\}$$

## Automorphisms

$$\begin{aligned}\mathrm{AutMan}_{\mathbb{C}}(\mathbb{C}\setminus\{0,1\}) &= \left\{f\in\mathrm{AutMan}_{\mathbb{C}}(\mathbb{C}\boldsymbol{P}^1)\,\middle|\, f\{0,1,\infty\}=\{0,1,\infty\}\right\}\\ &= \left\{z\mapsto z, 1-z, \frac{1}{z}, \frac{1}{1-z}, \frac{z}{z-1}, \frac{z-1}{z}\right\}\\ &\cong S_3\end{aligned}$$

An automorphism must be an *injective* holomorphic map

$$f:\mathbb{C}\setminus\{0,1\}\to\mathbb{C}\setminus\{0,1\}$$

with either poles or removable singularities at $0,1,\infty$

- Thus $f$ is an injective rational function.
- So it must be a Mobius transformation that sends $\{0,1,\infty\}$ to itself.

## universal covering

There is a holomorphic covering

$$D\to\mathbb{C}\setminus\{0,1\}$$

### $PSL(2)(\mathbb{Z})[2]\backslash H^2_{\mathrm{U}}$

$$\begin{array}{ccc} PSL(2)(\mathbb{Z})[2] & \curvearrowright & H^2_{\mathrm{U}}\\ & & \downarrow\\ & & \mathbb{C}\setminus\{0,1\}\end{array}$$

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Current note has 1 direct children and 1 total descendants.

- squishy
  - $\mathbb{C}\setminus\{0,1\}$, $PSL(2)(\mathbb{Z})[2]\backslash\mathbb{R}\boldsymbol{H}^2_{\mathrm{U}}$
    - Cohomology of holomorphic forms on $\mathbb{C}\setminus\{0,1\}$

And it has 30 siblings.

- squishy
  - $SL(2)(\mathbb{R})\curvearrowright\mathbb{R}^2$
  - $SL(2)(\mathbb{R})\curvearrowright H^2_{\mathrm{U}}$
  - $$\mathsf{Mat}_{\mathbb{C}}(2)$$
  - $A_r$, $\langle z\mapsto\lambda z\rangle\backslash\mathbb{R}\boldsymbol{H}^2_{\mathrm{U}}$
  - $\mathbb{C}\setminus\{0\}$, $\langle z\mapsto z+1\rangle\backslash\mathbb{R}\boldsymbol{H}^2_{\mathrm{U}}$
  - $\mathbb{C}\setminus\{0,1\}$, $PSL(2)(\mathbb{Z})[2]\backslash\mathbb{R}\boldsymbol{H}^2_{\mathrm{U}}$
  - $\mathbb{R}\ltimes_\theta\mathbb{C}^2$
  - $\boldsymbol{Cl}(\mathbb{R}^n,\langle\ ,\ \rangle_{0,n})$
  - $\mathbb{C}^n$
  - $U(\mathbb{C}\setminus\{0\})$
  - $PSL(2)(\mathbb{C})\curvearrowright\mathbb{C}P^1\cong_{\mathsf{Man}}S^2$
  - $\mathbb{C}P^n$
  - Farey graph, complex and tree
  - $GL$
  - Heisenberg group
  - Incomplete hyperbolic polygon mod side-pairing
  - $\mathbb{R}\boldsymbol{H}^2$, $\mathbb{R}H^2_{\mathrm{U}}$, $D^2$
  - $\mathbb{R}\boldsymbol{H}^n$
  - $D_n$ lattice
  - Matrices
  - $\mathsf{Mat}_{\mathbb{R}}(2)$
  - Mobius endomorphisms
  - Elementary group with finite and infinite sided Dirichlet polyhedra
  - Polynomials
  - $T^*S^1\not\cong_{\mathsf{Sym}}(\mathbb{R}^2\setminus\{0\},\mathrm{d}x\wedge\mathrm{d}y)$
  - $Z\left(\sum_{1\leq j\leq n}z_j^2-1\right)(\mathbb{C})\cong_{\mathsf{Sym}}T^*S^{n-1}$
  - $SL$

- $SO$
- Seifert–Weber dodecahedral 3-manifold
- $U(n, \mathbb{C})$