

This note, that [was found here](#), as a part of [a collection](#) is written (completely with human hands) by [Rupadarshi Ray](#), was created on December 8, 2025 1:52:23 AM, was last modified on April 30, 2026 11:57:21 PM, and was exported on September 7, 2026 2:50:35 PM.

Irreducible polynomials in $\mathbb{C}[x, y]$

Let

$$\mathbb{C}[x, y]_{\leq N}$$

be the $\mathbb{C}$ -vector space of all polynomials in x, y of degree at most N . It has $\mathbb{C}$ -dimension $\binom{N+2}{2}$.

Further, let

$$GL(2)(\mathbb{C}), \mathbb{C}^2$$

act on the "domain" of the polynomials by linear maps and translations. And let $\mathbb{C}^\times$ act on the vector space of polynomials by scaling.

Consider the quotient

$$\mathbb{C}^2, GL(2)(\mathbb{C}), \mathbb{C}^\times \backslash \mathbb{C}[x, y]_{\leq N}$$

(I should probably remove the zero polynomial from here, but it's okay!)

Bug

It has "dimension"

$$\binom{N+2}{2} - 2 - 4 - 1 = \binom{N+2}{2} - 7$$

homogenization

Consider the homogenization

$$\begin{aligned} \mathbb{C}[x, y]_{\leq N} &\rightarrow \mathbb{C}[x, y, z]_N \\ f(x, y) &\mapsto z^N f\left(\frac{x}{z}, \frac{y}{z}\right) \\ \sum_{i, j \leq N} c_{ij} x^i y^j &\mapsto \sum_{i, j \leq N} c_{ij} x^i y^j z^{N-i-j} \\ p(x, y, 1) &\leftarrow p(x, y, z) \end{aligned}$$

which is a $\mathbb{C}$ -linear bijection. This induces the map

$$\mathbb{C}^2, GL(2)(\mathbb{C}), \mathbb{C}^\times \backslash \mathbb{C}[x, y]_{\leq N} \rightarrow GL(3, \mathbb{C}) \backslash \mathbb{C}[x, y, z]_N$$

Bug

The map takes a space of dimension $\binom{N+2}{2} - 7$ to $\binom{N+2}{2} - 9$, so the "generic fibers" must be of dimension 2.

Current note has 0 direct children and 0 total descendants.

- [squishy](#)
 - [Polynomials](#)
 - [Irreducible polynomials in \$\mathbb{C}\[x, y\]\$](#)

And it has 17 siblings.

- [squishy](#)
 - [Polynomials](#)
 - $X^n + Y^n = Z^n$
 - $8X^3 + 4X^2 - 4X - 1$
 - $x^2 + y^2 - 3$
 - [Folium of Descartes](#)
$$X^3 + Y^3 - 3aXY$$
 - $X^3 + Y^3 - a$
 - $3X^3 + 10X^2 + 3 - Y^2$
 - $3X^8 + 10X^4 + 3 - Y^2$
 - $XYZ - 1$
 - [Pythagorean triples](#)
 - [Bolza curve](#)
 - [Irreducible polynomials in \$\mathbb{C}\[x, y\]\$](#)
 - [Polynomial maps \$\mathbb{C}^n \rightarrow \mathbb{C}^n\$ of degree at most \$d\$](#)
 - [Chebyshev polynomials of the first kind](#)
 - [Logistic map](#)
 - [Dynamics of the map \$z^2 + c\$](#)
 - $\begin{pmatrix} x \\ y \\ y \end{pmatrix} \mapsto \begin{pmatrix} (1+xy)^3z + y^2(1+xy)(4+3xy) \\ y + 3x(1+xy)^2z + 3xy^2(4+3xy) \\ 2x - 3x^2y - x^3z \end{pmatrix}$
 - $(w^2 - 1)^2$