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Open subsets of $\mathbb{R}^n$ equipped with the flat metric

Weyl's asymptotic law

Let $\Omega \subset \mathbb{R}^n$ is open with piecewise smooth boundary and with **Laplacian** $\Delta = -\sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}$ let

$$\begin{aligned}\Delta\varphi &= \lambda\varphi \\ \varphi|_{\partial\Omega} &= 0\end{aligned}$$

be the Helmholtz equation with Dirichlet boundary condition.

- the **spectrum**

$$\begin{aligned}\text{spec}(\Omega) &:= \sigma_{\text{p}}(\Delta : L^2(\Omega) \rightarrow L^2(\Omega)) \\ &= \{\lambda \mid \Delta\varphi = \lambda\varphi\}\end{aligned}$$

is known to be **discrete** with

$$0 < \lambda_1 \leq \lambda_2 \leq \cdots \rightarrow \infty$$

- the **eigenfunctions** of Δ form an **orthonormal basis** of $L^2(\Omega)$ (when normalized)

(Weyl's asymptotic law, 1911) Let $\Omega \subset \mathbb{R}^n$ is open with piecewise smooth boundary. Then element λ_k of spectrum of Ω grow like

$$\lambda_k \sim \frac{(2\pi)^2}{(m(D^n)m(\Omega))^{2/n}} k^{2/n} \text{ as } k \rightarrow \infty$$

So the spectrum determines the volume!

Definition. Isospectral open sets of $\mathbb{R}^n$

Two open sets $\Omega_1, \Omega_2 \subset \mathbb{R}^n$ are called **isospectral** if

$$\text{spec}(\Omega_1) = \text{spec}(\Omega_2)$$

and the multiplicity of each eigenvalue are also equal, that is for each $\lambda \in \text{spec}(\Omega_1) = \text{spec}(\Omega_2)$ we have

$$\dim \ker(\Delta \Big|_{L^2(\Omega_1)} - \lambda \text{Id}) = \dim \ker(\Delta \Big|_{L^2(\Omega_2)} - \lambda \text{Id})$$

Proposition: Isometric open sets are isospectral!

Current note has 0 direct children and 0 total descendants.

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And it has 39 siblings.

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- Discrete subgroups of $\mathbb{R}^n$
- Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- Ramified germs of smooth and holomorphic functions
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- ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in $\mathbb{R}^n$