ⓘ Info

This note, that was found here, as a part of a collection
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Sequence of functions from a metric space to $\mathbb{R}$

Definition. Sequence of functions from a metric space to $\mathbb{R}$

(X, d) be a metric space and $\forall n \in \mathbb{N}$

$$f_n : X \to \mathbb{R}$$

Then $(f_n)_{n\in\mathbb{N}}$ is a sequence of $\mathbb{R}$-valued functions on X

Definition. Point-wise convergence of sequence of functions from a metric space to $\mathbb{R}$

For a sequence of functions $X \to \mathbb{R}$

> **Definition.** Sequence of functions from a metric space to $\mathbb{R}$
>
> (X, d) be a metric space and $\forall n \in \mathbb{N}$
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> $$f_n : X \to \mathbb{R}$$
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> Then $(f_n)_{n\in\mathbb{N}}$ is a sequence of $\mathbb{R}$-valued functions on X

let $(f_n(x))_{n\in\mathbb{N}}$ converges for all $x \in X$ then (f_n) converges to

$$f(x) := \lim_{n\to\infty} f_n(x)$$

pointwise.

Definition. Point-wise convergence of series of functions from a metric space to $\mathbb{R}$

For a sequence of functions $X \to \mathbb{R}$

> **Definition.** Sequence of functions from a metric space to $\mathbb{R}$
>
> (X, d) be a metric space and $\forall n \in \mathbb{N}$
>
> $$f_n : X \to \mathbb{R}$$
>
> Then $(f_n)_{n\in\mathbb{N}}$ is a sequence of $\mathbb{R}$-valued functions on X

let $\sum_{n=1}^{\infty}(f_n(x))$ converges for all $x \in X$ then (f_n) converges to

$$\sum f(x) := \lim_{n\to\infty} f_n(x)$$

pointwise.

Definition. Uniform convergence of functions from a metric space to $\mathbb{R}$

For a sequence of functions $X \to \mathbb{R}$

> **Definition.** Sequence of functions from a metric space to $\mathbb{R}$
>
> (X, d) be a metric space and $\forall n \in \mathbb{N}$
>
> $$f_n : X \to \mathbb{R}$$
>
> Then $(f_n)_{n\in\mathbb{N}}$ is a sequence of $\mathbb{R}$-valued functions on X

then (f_n) **uniformly converges** to $f : E \to \mathbb{R}$ if $\forall\epsilon, \exists N_\epsilon \in \mathbb{N}$ such that

$$x \in E, n > N_\epsilon \implies |f_n(x) - f(x)| < \epsilon$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Metric spaces
    - Metric spaces
      - Function between metric spaces
        - to
          - Sequence of functions from a metric space to $\mathbb{R}$

And it has 2 siblings.

- structures based on sets
  - Metric spaces
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          - Functions from a metric space to $\mathbb{R}$
          - Sequence of functions from a metric space to $\mathbb{R}$