

Info

This note, that [was found here](#), as a part of [a collection](#) is written (completely with human hands) by [Rupadarshi Ray](#), was created on March 15, 2026 3:51:04 AM, was last modified on August 30, 2026 3:44:34 PM, and was exported on September 7, 2026 3:09:46 PM.

Finitely generated Fuchsian groups

↔

geometrically finite Fuchsian groups

cocompact

Let Γ be cocompact Fuchsian group. Then Γ has no parabolic elements.

💡 Consider

$$\begin{aligned} \varphi : H_{\mathbb{U}}^2 &\rightarrow [0, \infty) \\ z &\mapsto \min \{d(z, \gamma z) | \gamma \in \Gamma, o(\gamma) = \infty\} \end{aligned}$$

- Then $\varphi(z) = \varphi(\gamma z)$.
- ▮

 φ is continuous
- Thus image of φ is a compact subset of $(0, \infty)$.
- So there is a minimum of $\varphi > 0$.
- If $\gamma_0 \in \Gamma$ is parabolic, WLOG $\gamma(z) = z + 1$
- But then $ni \mapsto ni + 1$ has displacement $\leq \frac{1}{n}$.
- This contradicts existence of

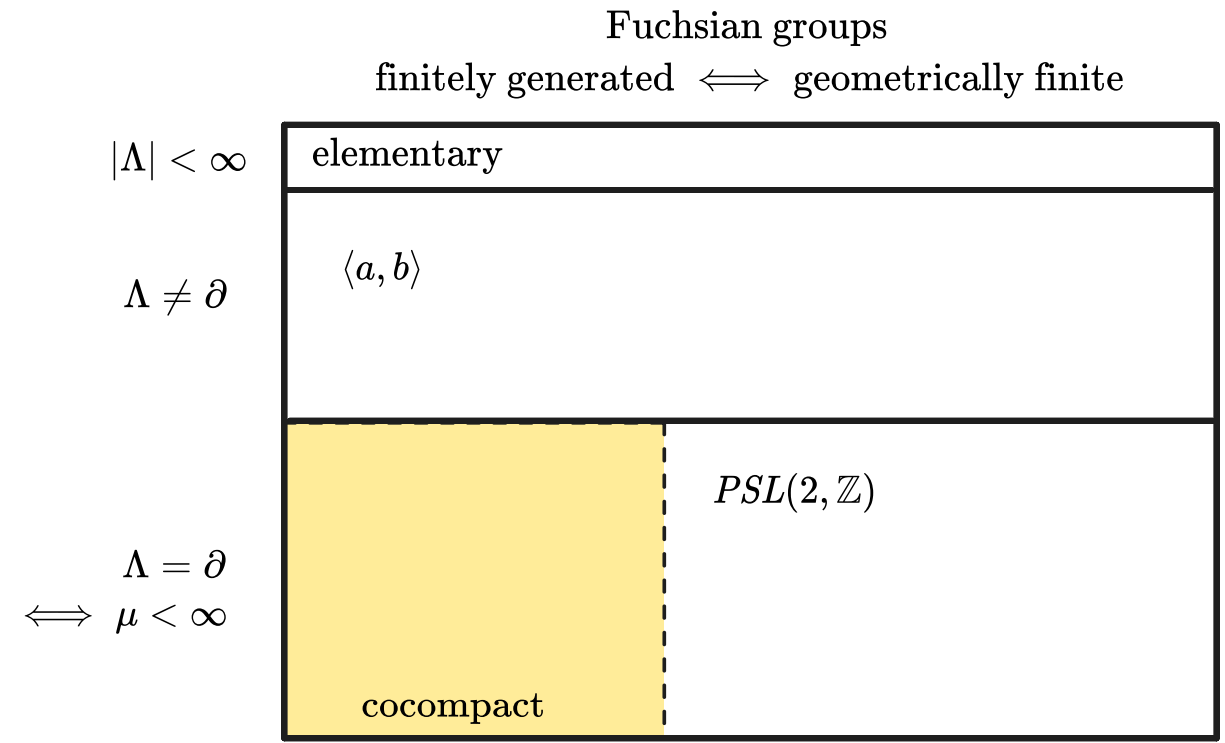
COROLLARY 4.2.7. *A Fuchsian group Γ is cocompact if and only if $\mu(\Gamma \backslash \mathbb{H}) < \infty$ and Γ contains no parabolic elements.*

geometrically finite

Definition. Definition

DEFINITION. A Fuchsian group Γ is called *geometrically finite* if there exists a convex fundamental region for Γ with finitely many sides.

finitely generated



Current note has 0 direct children and 0 total descendants.

stem

Objects stemming from $PSL(2)(\mathbb{R})$

Fuchsian groups $\Gamma <_d PSL(2)(\mathbb{R}) \leftrightarrow$ oriented hyperbolic 2-manifold/orbifolds $\rightarrow$ Riemann surfaces with branching signature

Finitely generated Fuchsian groups $\leftrightarrow$ geometrically finite Fuchsian groups

And it has 17 siblings.

stem

Objects stemming from $PSL(2)(\mathbb{R})$

Fuchsian groups $\Gamma <_d PSL(2)(\mathbb{R}) \leftrightarrow$ oriented hyperbolic 2-manifold/orbifolds $\rightarrow$ Riemann surfaces with branching signature

Artihmetic Fuchsian groups and Artihmetic hyperbolic 2-orbifolds

Cocompact Fuchsian groups

Central simple algebra

Elementary Fuchsian groups

Finitely generated Fuchsian groups $\leftrightarrow$ geometrically finite Fuchsian groups

Fuchsian groups of finite coarea

Geodesic flow of Fuchsian groups and hyperbolic surfaces

- [Decomposition of hyperbolic surfaces](#)
- [Infinitely generated Fuchsian groups](#)
- [Limit set of Fuchsian groups](#)
- [Marked hyperbolic surfaces](#)
- [subst.R PSL 2 discrete subg.over \$\mathbb{Q}\$ -bar](#)
- [Fuchsian Schottky groups](#)
- [Spectrum of hyperbolic surfaces](#)
- [Torsion-free Fuchsian groups](#)
- [Cocompact torsion-free Fuchsian groups](#)
- [Fuchsian triangle groups](#)