Info

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Pullback of a smooth vector bundle on a smooth manifold

Definition. Pullback of a smooth vector bundle on a smooth manifold

Let $E \to M$ be a smooth vector bundle and $g : N \to M$ a smooth map

$$\begin{tikzcd} & E \arrow[d, "\pi_E"] \\ N \arrow[r, "g"] & M \end{tikzcd}$$

Then the **pullback bundle** (pullback of E with g) is

$$g^*E := \{(x, v) \in N \times E | g(x) = \pi(v)\}$$

with $g^*E \to N$ as the projection onto the second factor and $g^*E \to E$ as projection onto the second factor such that

$$\begin{tikzcd} f^*E \arrow[r] \arrow[d] & E \arrow[d, "\pi_E"] \\ N \arrow[r, "g"] & M \end{tikzcd}$$

commutes.

Given any local frame of E

$$(\hat{\alpha}_i) : U_\alpha \to E$$

we have a local frame for g^*E

$$\begin{tikzcd} f^*E \arrow[r] & E \\ g^{-1}(U) \arrow[u, "\hat{\alpha}_i \circ g"] \arrow[r, "g"] & U_\alpha \arrow[u, "(\hat{\alpha}_i)"] \end{tikzcd}$$

Definition. Pullback of a connection on vector bundle

Let $E \to M$ be a smooth vector bundle with connection ∇ and $g : N \to M$ a smooth map. Then on the pullback bundle $N \to g^*E$

$$\begin{tikzcd} f^*E \arrow[r] \arrow[d] & E \arrow[d, "\pi_E"] \\ N \arrow[r, "g"] & M \end{tikzcd}$$

the **pullback connection** $g^*\nabla$ is the *unique connection* on g^*E such that for any section $s \in \Gamma E$

$$\begin{tikzcd} f^*E & E \\ N \arrow[u, "g^*s"] \arrow[r, "g"] & M \arrow[u, "s"] \end{tikzcd}$$

and $X \in TN$ we have

$$(g^*\nabla)_X(g^*s) = g^*(\nabla_{g_*X}(s))$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Vector bundle on smooth manifolds
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And it has 5 siblings.

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