Rigidity using measure homology

Theorem 11.8.4. *Let $M = B^n/\Gamma$ and $N = B^n/\mathrm{H}$ be compact orientable space-forms, with $n > 2$, and let $\tilde{\varphi} : B^n \to B^n$ be a lift of a smooth homotopy equivalence $\varphi : M \to N$. Then $\tilde{\varphi}_\infty : S^{n-1} \to S^{n-1}$ is a Möbius transformation.*

Proof: Let Δ be a hyperbolic, regular, ideal n-simplex in B^n with vertices $u_0, \ldots, u_n$. By Theorem 11.8.3, we have that $\tilde{\varphi}_\infty(u_0), \ldots, \tilde{\varphi}_\infty(u_n)$ are the vertices of a regular ideal n-simplex Δ' in B^n. Let f be the unique Möbius transformation of S^{n-1} such that $fu_i = \tilde{\varphi}_\infty(u_i)$ for each i. Then $f^{-1}\tilde{\varphi}_\infty(u_i) = u_i$ for each i.

Let g_i be the reflection of B^n in the side of Δ opposite the vertex u_i. Then the points $u_0, \ldots, u_{i-1}, g_iu_i, u_{i+1}, \ldots, u_n$ are the vertices of the regular ideal n-simplex $g_i\Delta$ in B^n. Consequently, the points

$$\tilde{\varphi}_\infty(u_0), \ldots, \tilde{\varphi}_\infty(u_{i-1}), \tilde{\varphi}_\infty(g_iu_i), \tilde{\varphi}_\infty(u_{i+1}), \ldots, \tilde{\varphi}(u_n)$$

are the vertices of a regular ideal n-simplex $(g_i\Delta)'$ in B^n. Let h_i be the reflection of B^n in the side of Δ' opposite the vertex $\tilde{\varphi}_\infty(u_i)$. By Lemma 13, we have that

$$(g_i\Delta)' = h_i\Delta'.$$

Therefore, we have

$$\tilde{\varphi}_\infty(g_iu_i) = h_i\tilde{\varphi}_\infty(u_i).$$

Hence

$$\begin{aligned} f^{-1}\tilde{\varphi}_\infty(g_iu_i) &= f^{-1}h_i\tilde{\varphi}_\infty(u_i) \\ &= f^{-1}h_iff^{-1}\tilde{\varphi}_\infty(u_i) \quad = \quad g_iu_i. \end{aligned}$$

Thus $f^{-1}\tilde{\varphi}_\infty$ fixes g_iu_i for each i.

Let G be the group generated by $g_0, \ldots, g_n$. By induction, $f^{-1}\tilde{\varphi}_\infty$ fixes each point of $U = \cup_{i=0}^n Gu_i$. Moreover, the set U is dense in S^{n-1} by Lemma 14. Therefore $f^{-1}\tilde{\varphi}_\infty$ is the identity map of S^{n-1} by continuity. Hence $\tilde{\varphi}_\infty = f$. Thus $\tilde{\varphi}_\infty$ is a Möbius transformation of S^{n-1}. □

Current note has 0 direct children and 0 total descendants.

- stem
  - O+ R1 n
    - Discrete subgroups of $O^+(1, n)$ ↔ hyperbolic n-manifolds/orbifolds
      - Rigidity of $\mathbb{R}$-hyperbolic manifolds
        - Rigidity using measure homology

And it has 3 siblings.

- stem
  - O+ R1 n
    - Discrete subgroups of $O^+(1, n)$ ↔ hyperbolic n-manifolds/orbifolds
      - Rigidity of $\mathbb{R}$-hyperbolic manifolds
        - Using ergodic current on $\partial^2\mathbb{R}\boldsymbol{H}^n$
        - Rigidity using measure homology
        - Rigidity of lattices using quasi-conformal mappings