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Path groupoid of a topological space

Definition. Category of paths in a topological space X

Let $X \in \mathbf{Top}$. Then define the category of paths upto reparameterization $\mathbf{Path}(X)$ with

- $objects$ are elements of X

$$\mathbf{Obj}(\mathbf{Path}(X)) = X$$

- $morphisms$ are paths upto reparameterization

$$\mathbf{HomPath}(x_0, x_1) := \frac{\{ \text{paths } p : [0, 1] \rightarrow X \mid p(0) = x_0, p(1) = x_1 \}}{p \sim \alpha \circ p \text{ for } \alpha : [0, 1] \rightarrow [0, 1], \alpha(0) = 0, \alpha(1) = 1}$$

- $morphism\ composition$ is concatenation of paths
- $identity\ morphism$

Fundamental path groupoid

Definition. Fundamental path groupoid of a topological space

Let $X \in \mathbf{Top}$. Then define the **path groupoid of X** , $\Pi_1(X)$ with

- $objects$ are the elements of X

$$\mathbf{Obj}(\Pi_1(X)) := X$$

- $morphisms$ are homotopy classes of paths relative to endpoints

$$\mathbf{Hom}\Pi_1(x_0, x_1) := \frac{\{ \text{paths } p : [0, 1] \rightarrow X : p(0) = x_0, p(1) = x_1 \}}{\text{homotopy rel } \{0, 1\}}$$

- $morphism\ composition$ is the [concatenation of paths](#) upto homotopy equivalence

$$[p_1] \circ [p_2] = [p_1 \cdot p_2]$$

Proposition:

Definition. Fundamental path groupoid of a topological space

Let $X \in \mathbf{Top}$. Then define the **path groupoid of X** , $\Pi_1(X)$ with

- $objects$ are the elements of X

$$\mathbf{Obj}(\Pi_1(X)) := X$$

- $morphisms$ are homotopy classes of paths relative to endpoints

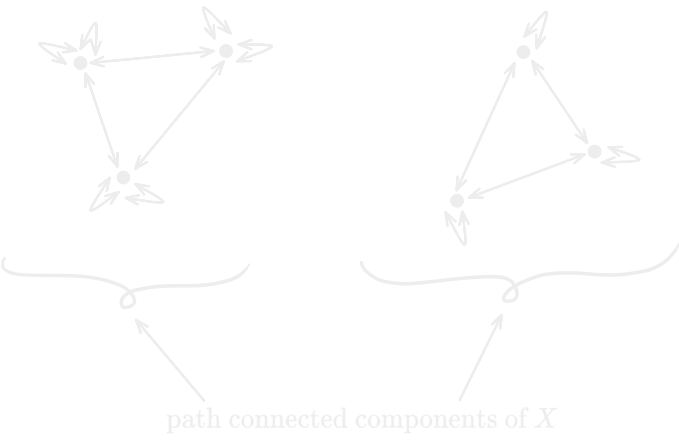
$$\mathbf{Hom}\Pi_1(x_0, x_1) := \frac{\{ \text{paths } p : [0, 1] \rightarrow X : p(0) = x_0, p(1) = x_1 \}}{\text{homotopy rel } \{0, 1\}}$$

- $morphism\ composition$ is the [concatenation of paths](#) upto homotopy equivalence

$$[p_1] \circ [p_2] = [p_1 \cdot p_2]$$

is a groupoid with

$$\mathbf{Id}_x = [c_x]$$



Current note has 0 direct children and 0 total descendants.

- [structures based on sets](#)
 - [Topological spaces](#)
 - [Paths in a topological space](#)
 - [Path groupoid of a topological space](#)

And it has 3 siblings.

- [structures based on sets](#)
 - [Topological spaces](#)
 - [Paths in a topological space](#)
 - [Concatenation of paths in a topological space](#)
 - [Fundamental group of a topological space](#)
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