Info

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$S_n \curvearrowright \mathbb{C}^n \to \mathbb{C}^n$

Definition. $S_n \curvearrowright \mathbb{C}^n$

$$\begin{gathered} S_n \curvearrowright \mathbb{C}^n \\ (z_1, z_2, \dots, z_n) \overset{\sigma}{\mapsto} (z_{\sigma(1)}, z_{\sigma(2)}, \dots, z_{\sigma(n)}) \end{gathered}$$

Viete map

We consider the map

$$\begin{gathered} v : \mathbb{C}^n \to \mathbb{C}^n \\ (a_1, a_2, \dots, a_n) \mapsto (c_1, \dots, c_n) \end{gathered}$$

such that

$$z^n + c_1 z^{n-1} + c_2 z^{n-2} + \dots + c_n = \prod_{i=1}^{n} (z - a_i)$$

Thus the map is given by

Definition. Viete map for $\mathbb{C}^n$

For $n \geq 1$ consider the **Viete map**

$$\begin{aligned} v : \mathbb{C}^n &\to \mathbb{C}^n \\ z &\mapsto (e_1(z), \dots, e_n(z)) \end{aligned}$$

where $e_1, \dots, e_n$ are elementary symmetric polynomials in $\mathbb{C}[z_1, \dots, z_n]$

> **Definition.** elementary symmetric polynomials in $k[X_1, \dots, X_n]$
>
> The elementary symmetric polynomials are
>
> $$\begin{aligned} e_1 &:= X_1 + X_2 + \dots + X_n \\ e_2 &:= X_1X_2 + X_2X_3 + X_3X_4 + \dots + X_{n-1}X_n \\ \dots \\ e_r &:= \sum_{1 \leq i_k < n} X_{i_1} X_{i_2} \dots X_{i_r} \\ \dots \\ e_n &:= X_1 X_2 \dots X_n \end{aligned}$$

> **Proposition:**
>
> $$v \circ \sigma = \sigma \circ v$$
>
> for all $\sigma \in S_n$

Viete map quotient S_n is a homeomorphism

using compactness

Current note has 0 direct children and 0 total descendants.

- squishy
  - $\mathbb{C}^n$
    - $S_n \curvearrowright \mathbb{C}^n \to \mathbb{C}^n$

And it has 3 siblings.

- squishy
  - $\mathbb{C}^n$
    - $GL(n, \mathbb{C}) \curvearrowright \mathbb{C}^n$
    - $S_n \curvearrowright \mathbb{C}^n \to \mathbb{C}^n$
    - Unit ball $D^n \subset \mathbb{C}^n$