Lebesgue measure on S^n

Definition.

$$m_{S^n}(E) := \frac{1}{n+1} m_{\mathbb{R}^{n+1}}\left(\left\{x \in B_1(0) \middle| \frac{x}{|x|} \in E\right\}\right)$$

We shall denote the unit sphere $\{x \in \mathbb{R}^n : |x| = 1\}$ by S^{n-1}. If $x \in \mathbb{R}^n \setminus \{0\}$, the **polar coordinates** of x are

$$r = |x| \in (0, \infty), \qquad x' = \frac{x}{|x|} \in S^{n-1}.$$

The map $\Phi(x) = (r, x')$ is a continuous bijection from $\mathbb{R}^n \setminus \{0\}$ to $(0, \infty) \times S^{n-1}$ whose (continuous) inverse is $\Phi^{-1}(r, x') = rx'$. We denote by m_* the Borel measure on $(0, \infty) \times S^{n-1}$ induced by Φ from Lebesgue measure on $\mathbb{R}^n$, that is, $m_*(E) = m(\Phi^{-1}(E))$. Moreover, we define the measure $\rho = \rho_n$ on $(0, \infty)$ by $\rho(E) = \int_E r^{n-1}\, dr$.

2.49 Theorem. *There is a unique Borel measure $\sigma = \sigma_{n-1}$ on S^{n-1} such that $m_* = \rho \times \sigma$. If f is Borel measurable on $\mathbb{R}^n$ and $f \geq 0$ or $f \in L^1(m)$, then*

$$\int_{\mathbb{R}^n} f(x)\, dx = \int_0^\infty \int_{S^{n-1}} f(rx')r^{n-1}\, d\sigma(x')\, dr. \tag{2.50}$$

Current note has 0 direct children and 0 total descendants.

- stretchy
  - S^n
    - Lebesgue measure on S^n

And it has 4 siblings.

- stretchy
  - S^n
    - $O(n+, \mathbb{R}) \curvearrowright S^n$
    - $U(n, \mathbb{C}) \curvearrowright S^{2n-1}$
    - Lebesgue measure on S^n
    - TS^n