Info

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Heisenberg group

#Wiki/group/Lie

Definition. Heisenberg group

The $2n+1$ dimensional Hisenberg group is

$$H(2n+1,\mathbb{R}) := \left\{ \begin{bmatrix} 1 & a_1 & \dots & a_n & c \\ 0 & & & & b_1 \\ 0 & & I_n & & \vdots \\ 0 & & & & b_n \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} \middle| a_1,\dots,a_n,c,b_1,\dots,b_n \in \mathbb{R} \right\}$$

which is a closed subgroup of $GL(n+2)$.

$$\mathfrak{H}(2n+1,\mathbb{R}) := \left\{ \begin{bmatrix} 1 & a_1 & \dots & a_n & c \\ 0 & & & & b_1 \\ 0 & & 0_n & & \vdots \\ 0 & & & & b_n \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} \middle| a_1,\dots,a_n,c,b_1,\dots,b_n \in \mathbb{R} \right\}$$

Proposition:

$$\exp : \mathfrak{H}(2n+1,\mathbb{R}) \to H(2n+1,\mathbb{R})$$

$$\exp \begin{bmatrix} 1 & a_1 & \dots & a_n & c \\ 0 & & & & b_1 \\ 0 & & 0_n & & \vdots \\ 0 & & & & b_n \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & a_1 & \dots & a_n & c+\frac{1}{2}\langle a,b\rangle_n \\ 0 & & & & b_1 \\ 0 & & I_n & & \vdots \\ 0 & & & & b_n \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

Current note has 2 direct children and 3 total descendants.

- squishy
  - Heisenberg group
    - $H(3,\mathbb{C})$
    - $H(3,\mathbb{R}) = N(3,\mathbb{R})$
      - $H(3,\mathbb{R})/\mathbb{Z}$

And it has 30 siblings.

- squishy
  - $SL(2)(\mathbb{R}) \curvearrowright \mathbb{R}^2$
  - $SL(2)(\mathbb{R}) \curvearrowright H^2_{\mathrm{U}}$
  - $\mathsf{Mat}_{\mathbb{C}}(2)$
  - $A_r, \langle z \mapsto \lambda z \rangle \backslash \mathbb{R}\boldsymbol{H}^2_{\mathrm{U}}$
  - $\mathbb{C}\setminus\{0\}, \langle z\mapsto z+1\rangle \backslash \mathbb{R}\boldsymbol{H}^2_{\mathrm{U}}$
  - $\mathbb{C}\setminus\{0,1\}, PSL(2)(\mathbb{Z})[2] \backslash \mathbb{R}\boldsymbol{H}^2_{\mathrm{U}}$
  - $\mathbb{R}\ltimes_\theta \mathbb{C}^2$
  - $\boldsymbol{Cl}(\mathbb{R}^n, \langle\ ,\ \rangle_{0,n})$
  - $\mathbb{C}^n$
  - $U(\mathbb{C}\setminus\{0\})$
  - $PSL(2)(\mathbb{C}) \curvearrowright \mathbb{C}P^1 \cong_{\mathsf{Man}} S^2$
  - $\mathbb{C}P^n$
  - Farey graph, complex and tree
  - GL
  - Heisenberg group
  - Incomplete hyperbolic polygon mod side-pairing
  - $\mathbb{R}\boldsymbol{H}^2, \mathbb{R}H^2_{\mathrm{U}}, D^2$
  - $\mathbb{R}\boldsymbol{H}^n$
  - D_n lattice
  - Matrices
  - $\mathsf{Mat}_{\mathbb{R}}(2)$
  - Mobius endomorphisms
  - Elementary group with finite and infinite sided Dirichlet polyhedra
  - Polynomials
  - $T^*S^1 \ncong_{\mathsf{Sym}} (\mathbb{R}^2\setminus\{0\}, \mathrm{d}x\wedge\mathrm{d}y)$
  - $Z\left(\sum_{1\le j\le n} z_j^2 - 1\right)(\mathbb{C}) \cong_{\mathsf{Sym}} T^*S^{n-1}$
  - SL
  - SO
  - Seifert–Weber dodecahedral 3-manifold
  - $U(n,\mathbb{C})$