Info

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Finitely generated, free Kleinian groups

$\leftrightarrow$

hyperbolic interior of 3-handlebodies

Theorem. *Let G be a function group. The following statements are equivalent.*
(i) G is a Schottky group.
(ii) The 2-complex K in the signature of G is an unmarked sphere.
(iii) Every structure subgroup of G is trivial.

Let $\Gamma <_d PSL(2)(\mathbb{C})$ be a Kleinian group with $\Lambda(\Gamma) \neq \partial \mathbb{R}\boldsymbol{H}^3$ which is free, of purely hyperbolic-type such that for each component Ω of $\Omega(\Gamma)$, $\mathrm{stab}_\Gamma(\Omega)\backslash^{\Omega}$ is a compact Riemann surfaces minus finitely many points. Then Γ is a Schottky group. In particular, Γ is finitely generated.

finitely generated, free, purely hyperbolic-type Kleinian group with $\Lambda(\Gamma) \neq S^2 \iff$ ultra-Schottky group

(Maskit, 1967) Let Γ be a Klienian group with $\Lambda(\Gamma) \neq \partial \mathbb{R}\boldsymbol{H}^3$. Then it is finitely generated, free of purely hyperbolic-type $\iff$ it is *ultra-Schottky*. In particular, Γ is convex cocompact.

proof for Fuchsian groups

- Let Γ be a finitely generated, free Fuchsian group.
  - $\implies$ it is geomet

    Let $\Gamma <_d PSL(2)(\mathbb{C})$ be a Kleinian group with $\Lambda(\Gamma) \neq \partial \mathbb{R}\boldsymbol{H}^3$ which is free, of purely hyperbolic-type such that for each component Ω of $\Omega(\Gamma)$, $\mathrm{stab}_\Gamma(\Omega)\backslash^{\Omega}$ is a compact Riemann surfaces minus finitely many points. Then Γ is a Schottky group. In particular, Γ is finitely generated.

    nitely many sides and no side is attached to itself
  - Does $D(\Gamma)$ have a side that is not a geodesic line??

proof for Klienian groups

- From

  Let $\Gamma <_d PSL(2)(\mathbb{C})$ be a Kleinian group with $\Lambda(\Gamma) \neq \partial \mathbb{R}\boldsymbol{H}^3$ which is free, of purely hyperbolic-type such that for each component Ω of $\Omega(\Gamma)$, $\mathrm{stab}_\Gamma(\Omega)\backslash^{\Omega}$ is a compact Riemann surfaces minus finitely many points. Then Γ is a Schottky group. In particular, Γ is finitely generated.

  and

  (Ahlfors finiteness theorem) Let Γ be a non-elementary finitely generated Kleinian group. Then

  $$\Gamma\backslash^{\Omega(\Gamma)}$$

  is a finite union of Riemann surfaces of finite type.

fg free, purely hyperbolic-type Kleinian group with limit set $= S^2$

The boundary

$$\partial S_g$$

consists of free, discrete groups that are ***not*** Schottky.
∂S_g contains a group with parabolic elements. It also contains a group of purely hyperbolic-type, which must have $\Lambda = S^2$.

[1]

Quotient of $\mathbb{R}\boldsymbol{H}^3$ by a finitely generated, free Kleinian group is homeomorphic to interior of a handlebody

- By

  (Marden's tameness conjecture turned theorem by Agol, Calegari-Gabai) Hyperbolic 3-manifolds with finitely generated fundamental groups are (*topologically tame*) homeomorphic to the interior of a compact 3-manifold possibly with boundary.

  we have

  $$G\backslash^{\mathbb{R}\boldsymbol{H}^3} \cong \mathrm{int}(N)$$

  where N is a compact 3-manifold with boundary.
- Hyperbolic 3-manifolds are irreducible $\implies$ $\mathrm{int}(N)$ irreducible $\implies$ N is irreducible $\implies$ N is prime.
- By

5.2. THEOREM. *Suppose* M *is a prime, compact 3-manifold with* $\pi_1(M)$ *a nontrivial free group. Then* M *is either*

(i) *a 2-sphere bundle over* S^1, *or*

(ii) *a (possibly nonorientable) cube with handles.*

we conclude N is an orientable cube with handles $\implies$ a handlebody H_g.

Current note has 0 direct children and 0 total descendants.

- stem
  - Objects stemming from $PSL(2)(\mathbb{C})$
    - Kleinian groups $\Gamma <_{\mathrm{d}} PSL(2)(\mathbb{C})$ $\leftrightarrow$ Hyperbolic 3-manifolds/orbifolds $_\Gamma\backslash^{\mathbb{R}}\boldsymbol{H}^3$
      - Finitely generated, free Kleinian groups

And it has 7 siblings.

- stem
  - Objects stemming from $PSL(2)(\mathbb{C})$
    - Kleinian groups $\Gamma <_{\mathrm{d}} PSL(2)(\mathbb{C})$ $\leftrightarrow$ Hyperbolic 3-manifolds/orbifolds $_\Gamma\backslash^{\mathbb{R}}\boldsymbol{H}^3$
      - Finitely generated, free Kleinian groups
      - 2-generated Kleinian groups
      - Finitely generated Kleinian groups $\leftrightarrow$ topologically tame oriented hyperbolic 3-manifolds/finite degree orbifold covers of
      - Building hyperbolic 3-manifolds by gluing polyhedra
      - Kleinian group with $\Lambda(\Gamma) \neq \partial\mathbb{R}\boldsymbol{H}^3$
      - Thick-think decomposition of hyperbolic 3-manifolds
      - Kleinian ultra-Schottky groups $\leftrightarrow$ convex cocompact hyperbolic interior of 3-handlebodies

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1. Michael Kapovich, Bernhard Leeb, 2017 - Discrete isometry groups of symmetric spaces

> If Γ is a Kleinian subgroup of $\mathbf{Mob}(\mathbb{S}^3)$, then the above results are not longer true, moreover, Γ can be geometrically infinite. For instance, take a free finitely generated purely hyperbolic discrete subgroup of $PSL(2,\mathbb{C})$, whose limit set is the 2-sphere (the existence of such groups was first established by V. Chuckrow [56]). The Möbius extension of this group to $\mathbb{S}^3$ has nonempty domain of discontinuity, but is not geometrically finite.

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