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## Holomorphic functions on a complex 1-manifold

If  $X$  is a compact connected Riemann surface, then

$$\mathcal{O}(X) \cong \mathbb{C}$$

Take any holomorphic  $f \in \mathcal{O}(X)$  then

- $|f|$  is a continuous function from compact to  $\mathbb{R}$ , so there exists a maximum, that is, there exists  $x_0 \in X$  such that

$$|f(x_0)| = \sup\{|f(x)| : x \in X\}$$

- $f^{-1}(x_0)$  is a closed subset of  $X$
- take  $x \in f^{-1}(x_0)$  and a chart nbd  $U_\alpha$  of  $x$  and move  $f|_{U_\alpha}$  onto a function between subsets of  $\mathbb{C}$ , then because [modulus of non-constant holomorphic functions cannot attain its maximum in the interior of its domain](#), it must be a constant on  $U_\alpha$ , which is a open nbd of  $x$ . So every element of  $f^{-1}(x_0)$  has a open nbd inside it. So  $f^{-1}(x_0)$  is **open**

Because  $X$  is connected,  $f^{-1}(x_0) = X$ . Thus  $f$  is constant on  $X$ .

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