Info

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Rigidity of discrete groups in topological groups

Definition. Local and global rigidity of homomorphisms

Let Γ be a finitely generated group and G be a topological group. Consider

$$\mathrm{HomGrp}(\Gamma, G)$$

with the compact-open topology.

ρ **is**	**if**	**that is**
locally rigid	there is a neighborhood U of ρ such that any $\rho' \in U$ is G-conjugate to ρ	$[\rho] \in {}^{\mathrm{HomGrp}(\Gamma,G)}/_{G}$ is isolated?
R-**globally rigid** for $R \subseteq \mathrm{HomGrp}(\Gamma, G)$	every $\rho' \in R$ is conjugate to ρ	$${}^{R}/_{G}$$ is a point
R-**globally** Aut-**rigid** for $R \subseteq \mathrm{HomGrp}(\Gamma, G)$		$$\mathrm{AutGrp}(\Gamma) \backslash {}^{R}/_{G}$$ is a point

Definition. Deformation rigidity

Let Γ be a finitely generated group and G be a path connected topological group. Then **deformation rigidty** of $\rho \in \mathrm{HomGrp}(\Gamma, G)$ means any continuous

$$\rho_t : (-\epsilon, \epsilon) \to \mathrm{HomGrp}(\Gamma, G)$$

such that $\rho_0 = \rho$ is G-conjugate to the constant path at ρ.

Definition. Strong rigidity

Let C be a subcategory of the category of homomorphisms

$$\rho : \Gamma \to G$$

where Γ is a discrete group and G is a topological group. Then $\rho \in \mathsf{C}$

- is **strongly rigid** in C if for any $\rho' : \Gamma' \to G' \in \mathsf{C}$, every isomorphism $\theta : \Gamma \cong \Gamma'$ extends to a unique isomorphism of topological groups $\Theta : G \cong G'$

$$\begin{tikzcd} \Theta: & G \arrow[r, phantom, "\cong"] & G' \\ \theta: & \Gamma \arrow[u, "\rho"] \arrow[r, phantom, "\cong"] & \Gamma' \arrow[u, "\rho'"] \end{tikzcd}$$

- is **superrigid** in C if any $\varphi : \Gamma \to G' \in \mathsf{C}$ extends to a continuous homomorphism

$$\Phi : G \to G'$$

- In particular, strongly rigid implies

$$\mathrm{AutGrp}(\Gamma) \cong_{\mathsf{Grp}} \mathrm{AutGrp}(G)$$

Superrigidity

Let Γ be a discrete group and G, G' be topological groups. Then a pair

$$\rho : \Gamma \to G \in \mathrm{HomGrp}(\Gamma, G)$$
$$R \subseteq \mathrm{HomGrp}(\Gamma, G')$$

(ρ, R) is **superrigid** if every homomorphism $\varphi : \Gamma \to G' \in R$ extends to a continuous homomorphism

$$\Phi : G \to G'$$

[1]

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Topological spaces
    - Topological groups
      - Rigidity of discrete groups in topological groups

And it has 7 siblings.

- structures based on sets
  - Topological spaces
    - Topological groups
      - Uniformly continuous functions on a topological group
      - Any onb of identity of a connected topological group generates it
      - Image of discrete subgroups of topological groups

- Rigidity of discrete groups in topological groups
- Subsets of topological groups
- Locally compact Hausdorff topological group
- Translation on a topological group

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1. Gromov - Rigidity of lattices, p.58 ↩