Info

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Convergent power series

Definition. Definition

Let

$$\sum_{n\in\mathbb{N}} a_n z^n$$

be a formal power series with coefficients in $\mathbb{C}$. We define

$$R(\underline{a}) := \begin{cases} +\infty & \text{when } |a_n|^{\frac{1}{n}} \xrightarrow{n\to\infty} 0 \\ \dfrac{1}{\limsup_{n\to\infty} |a_n|^{\frac{1}{n}}} & \\ 0 & \end{cases}$$

- Let $\underline{a}$ be a sequence.
  - For $z_0 \in \mathbb{C}\setminus\{0\}$, let the sequence $\left\{a_n z_0^n\right\}_{n\in\mathbb{N}}$ be bounded. Then we have

    $$\forall n \in \mathbb{N}, |a_n z_0^n| \leq K$$
    $$\forall n \in \mathbb{N}, |a_n|^{\frac{1}{n}} \leq \frac{K^{\frac{1}{n}}}{|z_0|}$$

    - So

      $$\limsup_{n\to\infty} |a_n|^{\frac{1}{n}} = \lim_{n\to\infty} \underbrace{\sup\left\{|a_n|^{\frac{1}{n}}\middle| n \geq m\right\}}_{\leq \frac{K^{\frac{1}{n}}}{|z_0|}}$$
      $$\leq \lim_{n\to\infty} \frac{K^{\frac{1}{n}}}{|z_0|}$$
      $$= \frac{1}{|z_0|}$$

    - Hence,

      $$0 < |z_0| \leq R(\underline{a})$$

  - Let $R(\underline{a}) > 0 \iff \exists r > 0$ with $R(\underline{a}) > r$.
    - Then

      $$\inf_{m\in\mathbb{N}} \sup\left\{|a_n|^{\frac{1}{n}}\middle| n \geq m\right\} < \frac{1}{r}$$

      meaning there is an m such that

      $$\sup\left\{|a_n|^{\frac{1}{n}}\middle| n \geq m\right\} < \frac{1}{r}$$
      $$\forall n \geq m, |a_n|^{\frac{1}{n}} < \frac{1}{r}$$
      $$\{a_n r^n\}_{n\in\mathbb{N}} \text{ is bounded}$$

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, {}_\Gamma\backslash^{\mathbb{C}^n}$
    - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - Equivalent descriptions of holomorphic functions
        - Convergent power series

And it has 4 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, {}_\Gamma\backslash^{\mathbb{C}^n}$
    - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - Equivalent descriptions of holomorphic functions
        - Cauchy integral formula for holomorphic functions
        - Derivative of holomorphic functions
        - Integral of holomorphic differential forms
        - Convergent power series