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Submanifolds of smooth manifolds

Definition. Embedded submanifolds of smooth manifolds

Let M be a smooth manifold with or without boundary.
A subset $S \subseteq M$ **embedded topological submanifold** if the inclusion

$$\iota : S \hookrightarrow M$$

is a *topological embedding*.

An **embedded smooth submanifold** of M is a subset $S \subseteq M$ which is

- a topological manifold without boundary in the *subspace* topology,
- endowed with a smooth structure
- with respect to which the inclusion

$$\iota : S \hookrightarrow M$$

is a *smooth embedding*.

Here, M is called the **ambient manifold** of S

- The **codimension** of S in M is $\dim M - \dim S$
- An **embedded topological/smooth hypersurface** is an embedded topological/smooth submanifold of codimension 1
- The embedded topological/smooth submanifold $S \subseteq M$ is **properly embedded** if the inclusion $\iota : S \hookrightarrow M$ is a proper map.

Proposition: Suppose M is a smooth manifold with or without boundary and $S \subseteq M$ is an *embedded submanifold*. Then S is **properly embedded** $\iff$ **it is a closed subset** of M .

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