Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on November 28, 2025 3:53:38 PM,
was last modified on August 28, 2026 3:08:36 PM, and
was exported on September 7, 2026 3:24:55 PM.

$\mathrm{EndTop}(X, Y)$

Let X be a topological space and (Y, d) be a metric space. Then the set of all continuous maps $X \to Y$

$$\mathrm{EndTop}(X, Y) \subseteq Y^X$$

compact-open topology on Y^X

Definition. Compact-open topology on Y^X

Let X be a topological space and (Y, d) be a metric space. Let $K \subseteq X$ be compact and $\epsilon > 0$ and consider the subsets

$$B_\epsilon(f, K) := \left\{ g \in Y^X \middle| \sup_{x \in K} d(f(x), g(x)) < \epsilon \right\}$$

The topology on Y^X generated by these subsets is the **compact-open** topology $\mathfrak{T}_{\text{cpt}}$.

Proposition: Consider

Definition. Compact-open topology on Y^X

Let X be a topological space and (Y, d) be a metric space. Let $K \subseteq X$ be compact and $\epsilon > 0$ and consider the subsets

$$B_\epsilon(f, K) := \left\{ g \in Y^X \middle| \sup_{x \in K} d(f(x), g(x)) < \epsilon \right\}$$

The topology on Y^X generated by these subsets is the **compact-open** topology $\mathfrak{T}_{\text{cpt}}$.

- The family

$$\left\{ B_\epsilon(f, K) \middle| f \in Y^X, K \subseteq_{\text{cpt}} X, \epsilon > 0 \right\}$$

forms a base for the compact-open topology.
- Let $f_n \in Y^X$ be a sequence and $f \in Y^X$. Then the convergence in compact open topology is equivalent to *uniform convergence on compact subsets* in X

$$\begin{aligned} & f_n \xrightarrow{\text{uniformly on compact sets in X}, n \to \infty} f \\ \iff & f_n \xrightarrow{(Y^X, \mathfrak{T}_{\text{cpt}})} f \end{aligned}$$

Let $f_1, f_2 \in Y^X$ and any $g \in B_{\epsilon_1}(f_1, K_1) \cap B_{\epsilon_2}(f_2, K_2)$.

- Set

$$\epsilon_0 := \min \left\{ \epsilon_1 - \sup_{x \in K_1} d(f_1(x), g(x)), \epsilon_2 - \sup_{x \in K_2} d(f_2(x), g(x)) \right\}$$

- Then

$$B_{\epsilon_0}(g, K_1 \cap K_2) \subset B_{\epsilon_1}(f_1, K_1) \cap B_{\epsilon_2}(f_2, K_2)$$

- Thus these sets form a base.
- Now

$$\begin{aligned} & f_n \xrightarrow{\text{uniformly on compact sets in } X, n \to \infty} f \\ \iff & \forall \epsilon > 0, K \subseteq_{\text{cpt}} X \exists N > 0 : \forall n > N \left(\underbrace{\sup_{x \in K} d(f_n(x), f(x)) < \epsilon}_{f_n \in B_\epsilon(f, K)} \right) \\ \iff & f_n \xrightarrow{(Y^X, \mathfrak{T}_{\text{cpt}})} f \end{aligned}$$

topologies on Y^X

We now have a few topologies on Y^X

$$\mathfrak{T}_{\text{product}} = \mathfrak{T}_{\text{pointwise}} \subset \mathfrak{T}_{\text{cpt}} \subset \mathfrak{T}_{\text{uniform}}$$

Proposition: If X is *compact* and (Y, d) is any metric space, then the compact-open topology on Y^X and the topology of uniform convergence coincides

$$\mathfrak{T}_{\text{cpt}} = \mathfrak{T}_{\text{uniform}}$$

when is $\mathrm{EndTop}(X, Y) \subseteq (Y^X, \mathfrak{T}_{\text{cpt}})$ closed

Proposition: If X is *locally compact* and (Y, d) is a metric space then

$$\mathrm{EndTop}(X, Y) \subseteq (Y^X, \mathfrak{T}_{\text{cpt}})$$

is closed.

[1]

Current note has 1 direct children and 1 total descendants.

- structures based on sets

- Topological spaces
  - $\mathrm{End}\mathsf{Top}(X, Y)$
    - Pre-compact subsets of $(\mathrm{End}\mathsf{Top}(X, Y), \mathfrak{T}_{\mathrm{cpt}})$

And it has 29 siblings.

- structures based on sets
  - Topological spaces
    - Group action on topological spaces
    - Discrete subgroup acting on coset space
    - Basis and subbasis for a topology
    - Principal G-bundles over topological spaces

$$G \curvearrowright P \to X$$

    - Vector bundles over topological spaces
    - Closure of a subset of topological space
    - Locally compact and 2-countable Hausdorff topological spaces are paracompact with countable refinement
    - Covering dimension of topological spaces
    - $\mathrm{End}\mathsf{Top}(X, Y)$
    - Eilenberg–Steenrod functors

$$(H_n : \mathsf{Top} \times \mathsf{Top} \to \mathsf{Ab}, \partial_n)$$

    - Local fields
    - Local fields of characteristic 0
    - Topological groups
    - Continuous group action on a topological space
    - Intersection of open, dense subsets of a topological space
    - Local topological spaces
    - Covering space of a topological space
    - Borel measures on a topological space
    - Convergence for nets in a topological space
    - Loops and a open cover of a topological space
    - Paths in a topological space
    - Product of topological spaces
    - Ringed topological spaces
    - Sheaf on open sets of a topological space
    - Closed subsets of a topological space
    - Discrete subsets of topological spaces
    - Types of topological spaces
    - Locally compact Hausdorff spaces
    - Topological vector spaces over $\mathbb{R}$ or $\mathbb{C}$

---

1. http://staff.ustc.edu.cn/~wangzuoq/Courses/20S-Topology/Notes/Lec14.pdf ↩