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Monotonic subsets in a measure space

- Let

$$A_1 \subseteq A_2 \subseteq \cdots \subseteq A_j \subseteq A_{j+1} \subseteq \cdots$$

is an **increasing** sequence of measurable sets.

- Then

$$\begin{aligned} \mu\left(\bigcup_{j \geq 1} A_j\right) &= \mu\left(\bigcup_{j \geq 1} (A_j \setminus (A_1 \cup \cdots \cup A_{j-1}))\right) \\ &= \sum_{j \geq 1} \mu(A_j \setminus (A_1 \cup \cdots \cup A_{j-1})) \\ &\triangleq \lim_{n \rightarrow \infty} \sum_{1 \leq j \leq n} \mu(A_j \setminus (A_1 \cup \cdots \cup A_{j-1})) \\ &= \lim_{n \rightarrow \infty} \mu\left(\bigsqcup_{1 \leq j \leq n} A_j \setminus (A_1 \cup \cdots \cup A_{j-1})\right) \\ &= \lim_{n \rightarrow \infty} \mu(A_n) \end{aligned}$$

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- [structures based on sets](#)
 - [Measure spaces](#)
 - [\[0, ∞\]-measure spaces](#)
 - [Monotonic subsets in a measure space](#)

And it has 4 siblings.

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 - [Measure spaces](#)
 - [\[0, ∞\]-measure spaces](#)
 - [Measurable action of a topological group on a positive measure space](#)
 - [Integral inequalities](#)
 - [Integration of measurable functions with respect to a \[0, ∞\]-measure](#)
 - [Monotonic subsets in a measure space](#)