Info

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Continuous functions between metric spaces

A *discontinuity* at $p \in X$ is described by

$$\exists \epsilon > 0 : \forall \delta > 0, d_X(x,p) < \delta \implies d_Y(f(x), f(p)) > \epsilon$$

that is, there exists a fixed $\epsilon > 0$ such that for all

Definition. Continuous functions between metric spaces

A function between two metric spaces $f : X \to Y$ is **continuous at** $p \in X$ if $\forall \epsilon, \exists \delta_\epsilon$ satisfying

$$\begin{aligned} &\forall x \in X, d_X(x,p) < \delta_\epsilon \text{ we have } d_Y(f(x), f(p)) < \epsilon \\ \iff & f(B(\delta_\epsilon, x)) \subseteq B(\epsilon, f(x)) \end{aligned}$$

If f is continuous at all $p \in E$, then f is continuous on X.

$$\begin{aligned} & B(\delta, x) && \mapsto B_\epsilon(f(x)) \\ \text{or } & f(B(\delta, x)) && \subseteq B_\epsilon(f(x)) \\ \text{or } & B(\delta, x) && \subseteq \operatorname{preim}_f(B(\epsilon, f(x))) \end{aligned}$$

If $p \in X$ is not a limit point of X.

Given a function between two metric spaces $f : X \to Y$ and $p \in X$ is a limit point of X then

$$f \text{ is continuous} \iff \lim_{x \to p} f(x) = f(p)$$

A function between two metric spaces $f : X \to Y$ is continuous at x

$$\iff$$

for any neighbourhood $N_{f(p)} \subseteq Y$ of $f(x)$

$$\operatorname{preim}_f(N_{f(x)}) \ (\subseteq X) \text{ is a neighbourhood of } x$$

A function between two metric spaces $f : X \to Y$ is continuous, that is,

Definition. Continuous functions between metric spaces

A function between two metric spaces $f : X \to Y$ is **continuous at** $p \in X$ if $\forall \epsilon, \exists \delta_\epsilon$ satisfying

$$\begin{aligned} &\forall x \in X, d_X(x,p) < \delta_\epsilon \text{ we have } d_Y(f(x), f(p)) < \epsilon \\ \iff & f(B(\delta_\epsilon, x)) \subseteq B(\epsilon, f(x)) \end{aligned}$$

If f is continuous at all $p \in E$, then f is continuous on X.

$\iff$ $\forall U \subseteq Y$ open subset, the preimage $f^{-1}(U) \subseteq X$ is open.

Equivalent to

$$V \subseteq Y \text{ is closed in } Y \implies f^{-1}(V) \text{ is closed in } X$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Metric spaces
    - Metric spaces
      - Function between metric spaces
        - Continuous functions between metric spaces

And it has 6 siblings.

- structures based on sets
  - Metric spaces
    - Metric spaces
      - Function between metric spaces
        - Continuous functions between metric spaces
        - Uniformly continuous functions on a metric space
        - Contraction map in a metric space
        - Topological entropy of maps between metric spaces
        - Limit of a function between metric spaces
        - Sensitive dependence on initial conditions of dynamics of a map between metric space