Info

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Lebesgue density of measurable sets

Definition. Density of measurable sets

Let $E \subseteq \mathbb{R}^d$ be a measurable set. Then

$$\operatorname{density}_E(x) := \lim_{\epsilon \to 0} \frac{m(B_\epsilon(x) \cap E)}{m(B_\epsilon(x))}$$

is the **density** of E at $x \in \mathbb{R}^d$. The **density** set of E is the set of all points where the density of E is 1

$$\operatorname{dens}(E) := \left\{ x \in E \middle| \lim_{x \in B; m(B) \to 0} \frac{m(B \cap E)}{m(B)} = 1 \right\}$$

(Lebesgue density theorem) For almost every $x \in E$, we have

$$\operatorname{density}_E(x) = 1$$

using differentiation

(Differentiation of the Lebesgue integral on balls) If $f \in L^1_{\text{loc}}(\mathbb{R}^d)$ then

$$\lim_{r \to 0} (A_r f)(x) = f(x) \text{ aeon } x \in \mathbb{R}^d$$

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, {}_\Gamma\backslash^{\mathbb{C}^n}$
    - Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - Lebesgue density of measurable sets

And it has 15 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, {}_\Gamma\backslash^{\mathbb{C}^n}$
    - Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - Functions with uniformly bounded mean oscillations on cubes
      - Decomposition of $f \in L^p$ into bounded and unbounded parts
      - Calderon-Zygmund decomposition
      - Lebesgue density of measurable sets
      - Functions with uniformly bounded mean oscillations on dyadic cubes
      - Volterra operator $\int_{[0,-]} : L^1_{\text{loc}}[0,1) \to \mathcal{C}[0,1)$
      - Measurable functions on $\mathbb{R}^n$
      - (ϵ, n)-measurable function
      - Integrability and integral of measurable functions on $\mathbb{R}^n$
      - Hardy-Littlewood maximal functions of $L^1_{\text{loc}}(\mathbb{R}^n)$
      - Lebesgue averaging and differentiation
      - Integrals of a monotonically converging sequence of functions
      - Lebesgue integral from measure of undergraph
      - $L^{p,q}$
      - $\mathrm{E}([0,1]) = \mathrm{E}(0,1), \mathrm{E}([-\pi,\pi]) = \mathrm{E}(S^1)$