ⓘ Info

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Algebra of subsets of a set

Definition. A σ-algebra

An **algebra** $\mathfrak{M}$ of subsets of X

$$\mathfrak{M} \subseteq 2^X$$

satsifies

- $X \in \mathfrak{M}$
- *(closed in taking complements)* $S \in \mathfrak{M} \implies X \setminus S \in \mathfrak{M}$
- *(closed in finite unions)* if $S_1, S_2 \in \mathfrak{M}$ then $S_1 \cup S_2 \in \mathfrak{M}$

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- $X \in \mathfrak{M}$
- *(closed in taking complements)* $S \in \mathfrak{M} \implies X \setminus S \in \mathfrak{M}$
- *(closed in countably infinite unions)* if $S_i \in \mathfrak{M}$ for all $i \in \mathbb{N}$ then

$$\bigcup_{i \in \mathbb{N}} S_i \in \mathfrak{M}$$

- By $X \in \mathfrak{M}$ and property of being closed in taking complements imply

$$\emptyset \in \mathfrak{M}$$

- By taking $S_i = \emptyset$ for $i \geq N+1$ we have

$$\bigcup_{0 \leq i \leq N} S_i \in \mathfrak{M}$$

  thus the σ-algebra is *closed under taking finite unions.*
- Since

$$\bigcap_{n \geq 1} S_n = \left(\bigcup_{n \geq 1} S_n^{\mathsf{c}} \right)^{\mathsf{c}}$$

  a σ-algebra is closed under *countable (finite or infinite) intersections.*
- Since

$$A \setminus B = B^{\mathsf{c}} \cap A$$

  thus $A, B \in \mathfrak{M}$ implies $A \setminus B \in \mathfrak{M}$.

We conclude a σ-algebra has the structure

$$(\mathfrak{M}, \cap, \cup, \setminus)$$

> **Proposition:** Let X be a set and $A \subset 2^X$. Then there exists a smallest σ-algebra $\mathfrak{M}$ in 2^X such that $A \subset \mathfrak{M}$.

Consider

$$\mathfrak{M} := \bigcap_{A \subset B \ \sigma\text{-algebra in } 2^X} B$$

...

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Measure spaces
    - Algebra of subsets of a set

And it has 7 siblings.

- structures based on sets
  - Measure spaces
    - Algebra of subsets of a set
    - Meas
    - Functions on measure spaces
    - $\mathbb{C}$-measures
    - $[0, \infty]$-measure spaces
    - Measure preserving endomorphisms
    - $[0, 1]$-measure spaces