Info

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$PSL(2)(\mathbb{Z})[2] := \ker(PSL(2)(\mathbb{Z}) \to PSL(2)(\mathbb{Z}_2)) <_{\mathsf{d}} PSL(2)(\mathbb{Z}) <_{\mathsf{d}} PSL(2)(\mathbb{R})$

We have

$$PSL(2)(\mathbb{Z})[2] = \left\{ \begin{bmatrix} \text{odd} & \text{even} \\ \text{even} & \text{odd} \end{bmatrix} \in PSL(2)(\mathbb{Z}) \right\}$$

```
G = Gamma(2)
print(G.image_mod_n())
print(G.ncusps())
```

> **Proposition:**
>
> $$PSL(2)(\mathbb{Z})[2] = \left\langle \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \right\rangle$$

```
FareySymbol(Gamma(2)).fundamental_domain()
```

its free!

> **Proposition:** The group
>
> $$\left\langle \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \right\rangle \curvearrowright \mathbb{Z}^2$$
>
> by linear maps plays ping-pong on
>
> $$A := \left\{ (x, y) \in \mathbb{Z}^2 \middle| |y| > |x| \right\}$$
> $$B := \left\{ (x, y) \in \mathbb{Z}^2 \middle| |x| > |y| \right\}$$
>
> Therefore, by
>
> **(Ping-pong lemma for F_2)** Let $\Gamma = \langle a, b \rangle$ for $a, b \in \Gamma$ and there is a action $\Gamma \curvearrowright X$ such that there are non-empty $A, B \subseteq X$ with $B \nsubseteq A$ such that for all $n \in \mathbb{Z} \setminus \{0\}$ we have
>
> $$a^n(B) \subseteq A, b^n(A) \subseteq B$$
>
> Then $\Gamma \cong F_2$ and Γ is freely generated by $\langle a, b \rangle$.
>
> it is free.

Let $(x, y) \in \mathbb{Z}^2$ such that $|x| > |y|$. [1]

- Then

  $$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^n \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 2nx + y \end{bmatrix}$$

  and

  $$|2nx + y| \geq |2n| |x| - |y| \geq 2 |x| - |y| > |x|$$

  Thus $a^n(A) \subseteq A$.
- ...

$PSL(2)(\mathbb{Z})[2] \backslash H^2_{\mathrm{U}}$

$$\begin{array}{ccc} PSL(2)(\mathbb{Z})[2] & \curvearrowright & H^2_{\mathrm{U}} \\ & & \downarrow \\ & & \mathbb{C} \setminus \{0, 1\} \end{array}$$

Current note has 0 direct children and 0 total descendants.

- squishy
  - SL
    - $PSL(2)(\mathbb{Z})[2]$

And it has 15 siblings.

- squishy
  - SL
    - SL(2)
    - $SL(2)(\mathbb{C})$
    - $PSL(n, \mathbb{C})$
    - $SL(2)(\mathbb{R})$
    - $SL(2)(\mathbb{R}) / SL(2)(\mathbb{Z})$
    - H_q
    - $PSL(2)(\mathbb{Z})[2]$
    - $[PSL(2)(\mathbb{Z})[2], PSL(2)(\mathbb{Z})[2$]]
    - $PSL(2)(\mathbb{Z})[N]$
    - $SL(3, \mathbb{R})$
    - $SL(2)(\mathbb{Z})$
    - $PSL(2)(\mathbb{Z})$
    - Congruent subgroups of $PSL(2)(\mathbb{Z})$

- $SL(n, \mathbb{Z})$
- $SL(2)(\mathbb{Z}_2)$

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1. Sanov subgroup in SL(2,Z) is free of rank two - Groupprops (subwiki.org) ↩