Info

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$\mathrm{HomGrp}(F_2, G)$

- Let $G \leq GL(n,\mathbb{R})$ be Zariski closed. Of course

$$\mathrm{HomGrp}(F_2, G) \cong_{\mathsf{Top},\mathsf{An}_{\mathbb{R}},\mathsf{Aff}_{\mathbb{R}}} G \times G$$

- Let $w(a,b) \in F_2 := \langle a,b \rangle$ be a (reduced) word. Then this gives a map

$$\begin{aligned} w : G \times G &\to G \\ (x,y) &\mapsto w(x,y) \end{aligned}$$

  which is a *polynomial* map.

  - Consider the fiber

  $$w^{-1}(I)$$

    which is either $G \times G$ or a Zariski closed subset of *lower dimension*.
  - Then

  $$\{(x,y) \in G \times G | \langle x,y \rangle \ncong F_2\} = \bigcup_{w \in F_2 \setminus \{i\}} w^{-1}(I)$$

    is a countable union of such Zariski closed subsets, or is $G \times G$.

Thus,

> **Proposition:** Let $G \leq GL(n,\mathbb{R})$ be a Zariski closed subgroup. Consider the set of all pairs that do not generate F_2
>
> $$\{(x,y) \in G \times G | \langle x,y \rangle \ncong F_2\}$$
>
> is either
>
> - $G \times G$
> - or (countable union of Zariski closed proper subsets) a meagre subset of Haar measure zero.

Current note has 0 direct children and 0 total descendants.

- stem
  - HomGrp
    - $\mathrm{HomGrp}(F_2, G)$

And it has 1 siblings.

- stem
  - HomGrp
    - $\mathrm{HomGrp}(F_2, G)$