Info

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Potential flows

Definition. Potential flows

$$(M, g, V)$$

Let (M, g) be a semi-Riemannian manifold and

$$V : M \to \mathbb{R}$$

be a smooth function. Then the potential flow of V on (M, g) is the flow on T^*M associated with the Hamiltonian vector field of

$$H(P) := \frac{1}{2}\|P\|_g^2 + V(\pi(P))$$

with respect to the usual symplectic structure.

- The equation on M is

$$\nabla_{\dot\gamma}\dot\gamma = -\mathrm{grad}(V)$$

- $V = 0$ reproduces **geodesic flows**.
- $$\begin{array}{ccccc} \mathcal{C}^\infty(M) & \to & \mathcal{C}^\infty(T^*M) & \to & \mathrm{Vec}(T^*M) \\ V & \mapsto & \frac{1}{2}\|p\|_g^2 + V \circ \pi & & \end{array}$$

potential flow as geodesic flow

On the open subset $U := V^{-1}(-\infty, E) \subseteq M$, consider the metric

$$h_E := 2(E - V)g\Big|_U$$

Geodesic flow for this metric corresponds to Hamiltonian vector field of

$$\frac{1}{2}\|p\|_h^2 = \frac{4}{2}(E - V)^2\|p\|_g^2$$

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And it has 2 siblings.

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