ⓘ Info

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Classification of Lie groups

Let $\mathfrak{g}$ be a Lie $\mathbb{R}$-algebra. There is an **unique** simply connected Lie group G with Lie algebra $\mathrm{Lie}(G) \cong \mathfrak{g}$.
The connected Lie groups whose Lie algebras are isomorphic to $\mathfrak{g}$ are precisely

$$\frac{G}{\Gamma}$$

where $\Gamma \lhd G$ is a **discrete normal subgroup**.
Arbitrary Lie groups Q are extensions of connected Lie groups by discrete Lie groups

$$0 \to Q_0 \hookrightarrow Q \twoheadrightarrow \pi_0(Q) \to 0$$

Theorem 21.32. *Let $\mathfrak{g}$ be a finite-dimensional Lie algebra. The connected Lie groups whose Lie algebras are isomorphic to $\mathfrak{g}$ are (up to isomorphism) precisely those of the form G/Γ, where G is the simply connected Lie group with Lie algebra $\mathfrak{g}$, and Γ is a discrete normal subgroup of G.*

21-22. If G and H are Lie groups, and there exists a surjective Lie group homomorphism from G to H with kernel G_0, we say that G is an ***extension of G_0 by H***. Let $\mathfrak{g}$ be any finite-dimensional Lie algebra, and show that the disconnected Lie groups whose Lie algebras are isomorphic to $\mathfrak{g}$ are precisely the extensions of the connected ones by discrete Lie groups. [Hint: see Proposition 7.15.] (*Used on p. 557.*)

Current note has 1 direct children and 1 total descendants.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold group (Lie groups)
      - Classification of Lie groups
        - 2 dimensional Lie groups

And it has 18 siblings.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold group (Lie groups)
      - Adjoint representation of a Lie group
      - Conjugation action of a Lie group on itself
      - Classification of Lie groups
      - Compact Lie groups
      - Connected Lie groups with discrete center
      - Connected Lie groups with non-surjective exponential map
      - Discrete subgroups of Lie groups
      - Smooth maps onto a Lie group
      - Lie groups with surjective exponential map
      - Maurer-Cartan form
      - Left-invariant Lagrangian on a Lie group
      - Lie algebra of a lie group
      - Bi-invariant metrics on Lie groups
      - Left invariant Riemannian metric on a Lie group
      - Semi-simple $\mathbb{R}$-Lie groups
      - Almost simple groups
      - Virtual subgroups of Lie groups
      - Lie group with dilations