Info

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Cotangent bundle T^*M on a smooth manifold M

Definition. Cotangent bundle T^*M on a smooth manifold M

> With local coordinates $x : U \subseteq M \to \mathbb{R}^{\dim M}$ we have induced **local coordinates**
>
> $$\begin{aligned}(x, \xi) : U \times \pi^{-1}(U) &\to \mathbb{R}^{2\dim M}\\ \left(q, \sum_i \xi_i \mathrm{d}x_i\right) &\mapsto (x(q), \xi_1, \ldots, \xi_n)\end{aligned}$$

tautological 1-form

Definition. Tautological 1-form on T^*M

We have the projection smooth map

$$\pi : T^*M \to M$$

then its derivative is the pushforward

$$\pi_* : TT^*M \to TM$$

whose dual is the *pullback*

$$\pi^* : T^*M \to T^*T^*M$$

which means it is a 1-form on T^*M called the **tautological 1-form**.

The 2-form

$$\omega := -\mathrm{d}\pi_*$$

is a symplectic form on T^*M

- On

  we have

  $$(\pi^*)_{(p, \sum_i \xi_i \mathrm{d}x_i)} = \sum_i \xi_i \mathrm{d}x_i$$

  and

  $$\omega = \sum_i \mathrm{d}x_i \wedge \mathrm{d}\xi_i$$

(Reproducing property) Let

$$\mu : M \to T^*M$$

be a smooth 1-form on M, which in particular is a smooth map. Then the pullback of the tautological 1-form is

$$\mu^*(\pi^*) = \mu$$

Conversely if $\alpha \in \Omega^1(T^*M)$ such that $\mu^*\alpha = \mu$ for all $\mu \in \Omega^1(M)$ then α is the tautological 1-form.

$$\mu^*\pi^* = (\pi \circ \mu)^*(\mu)$$

$$\begin{aligned}\mathrm{Vec}(M) &\to \mathcal{C}^\infty(T^*M)\\ X &\mapsto (p \mapsto p(X))\end{aligned}$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Tangents on a smooth manifold
        - Tangent space of a smooth manifold
          - Cotangent bundle T^*M on a smooth manifold M

And it has 8 siblings.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Tangents on a smooth manifold
        - Tangent space of a smooth manifold
          - Canonical bijection of T_pM with equivalence classes of velocities
          - Tangents on smooth manifolds as derivations $\mathcal{C}^\infty(M) \to \mathbb{R}$
          - local basis of a tangent space of a smooth manifold
          - Tangent bundle TM of a smooth manifold M
          - Cotangent bundle T^*M on a smooth manifold M
          - Comparison of constructions with tangent vectors

- Tangent frames on smooth manifolds
- Tangent vector fields on a smooth manifold