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Connected Lie groups with non-surjective exponential map

Proposition: There is *no* Riemannian metric on a Lie group G with non-surjective exponential map such that

$$\exp_{i_G} = \exp : \text{Lie}(G) \rightarrow G$$

Let there is such a metric, then by

(Hopf-Rinow) Let (M, g) be a connected Riemannian manifold. Then M is geodesically complete

(M, d_g) is a complete metric space

$\exp_p : T_p M \rightarrow M$ is defined for some $p \in M$

every closed d -bounded set in M is compact.

If (M, g) satisfies any (hence all) of the above, then each pair of points in M can be joined by a **length-minimizing** geodesic segment. In particular, for each $p \in M$ the exponential

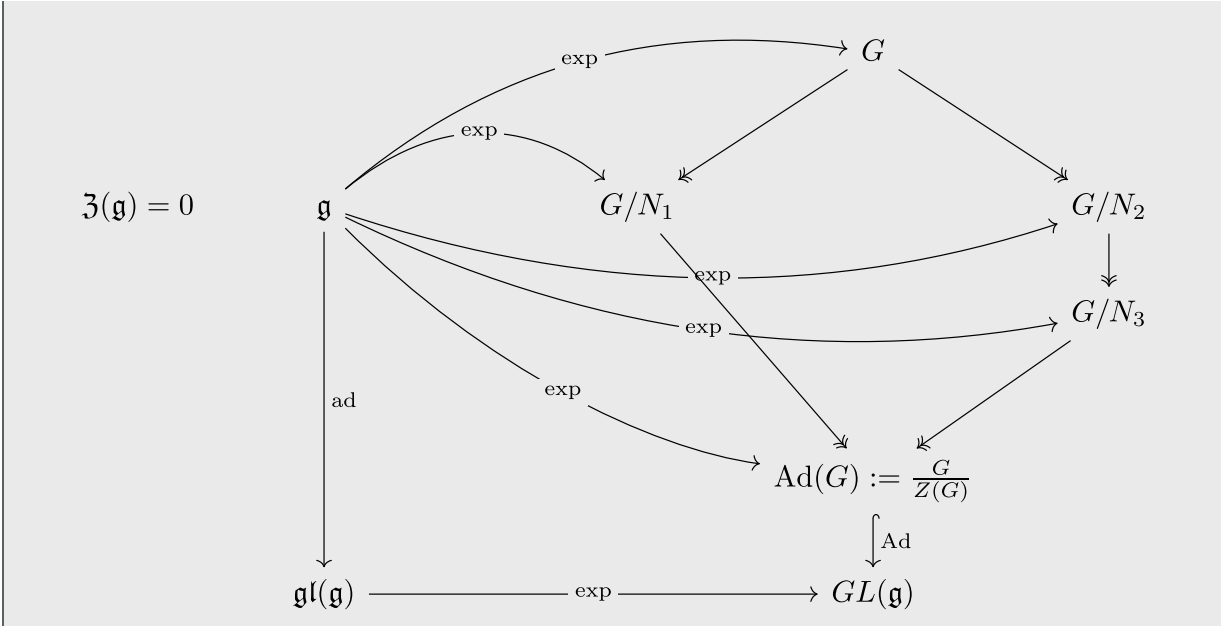
$\exp_p : T_p M \rightarrow M$

is surjective.

it is complete, hence $\exp = \exp_{i_G}$ must be surjective.

Covering of a *connected Lie group with non-surjective exponential map*

Connected Lie groups with non-surjective exponential map and discrete center



- If exponential is *not* surjective onto $\text{Ad}(G)$ then it is not surjective onto any Lie group with Lie algebra $\mathfrak{g}$.

Current note has 0 direct children and 0 total descendants.

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And it has 18 siblings.

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