Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
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Homeomorphisms $S^1 \to S^1$

For any homeomorphism

$$f : S^1 \to S^1$$

there exists a lift

$$F : \mathbb{R} \to \mathbb{R},\ \pi_{\mathbb{R}/\mathbb{Z}} \circ F = f \circ \pi_{\mathbb{R}/\mathbb{Z}}$$

where for any such lift F, F is an homeomorphism and any two such lift F, G is separated by an integer

$$F - G = k \in \mathbb{Z}$$

orientation preserving

Theorem

rotation number

From

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$$F - G = k \in \mathbb{Z}$$

given a lift F of orientation-preserving homeomorphism

$$f : S^1 \to S^1$$

, for every $x \in \mathbb{R}$ the limit

$$\lim_{n\to\infty} \frac{F^n(x) - x}{x}$$

exists, is *independent of* x and does not depend on the choice of lift of f.

Definition. Rotation number for a orientation-preserving circle homeomorphism

From

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for orientation-preserving homeomorphism

$$f : S^1 \to S^1$$

we this number to be the **rotation number** denoted by

$$\mathrm{rotation}(f)$$

Current note has 0 direct children and 0 total descendants.

- stretchy
  - S^1
    - Homeomorphisms $S^1 \to S^1$

And it has 4 siblings.

- stretchy
  - S^1

- The map $z \mapsto z^n$ on the circle
- Left translation $S^1 \curvearrowright S^1$
- Homeomorphisms $S^1 \to S^1$
- Loops in S^1 upto homotopy